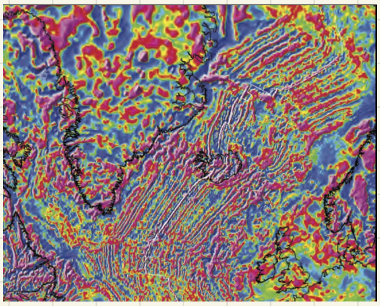
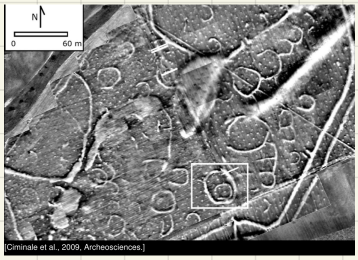
Magnetic Method

- linear features in the oceans
  - → seafloor spreading and plate tectonics
  - → "mono" of magnetics



- archeological sights
- bombs
- comparatively easy and cheap



[Ciminale et al., 2009, Archeosciences.]

Goal: Interpret deviations / anomalies of the Earth's magnetic field and to link to variability in the sub-surface.
It's a passive method. ("just Earth's magnetic field")

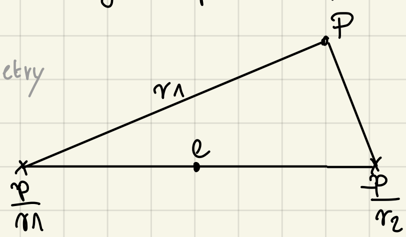
Governing equations: Dipole field (magnetic)

Classic derivation of the Dipole field:

① There are no magnetic monopoles
→ simplest unit is dipole field

Imagine two magnetic poles (N, S) for magnetic potential A

- radial symmetry
- decay $\sim \frac{1}{r}$



cf.

"homogenous half space"
$U \sim \frac{A}{r}$

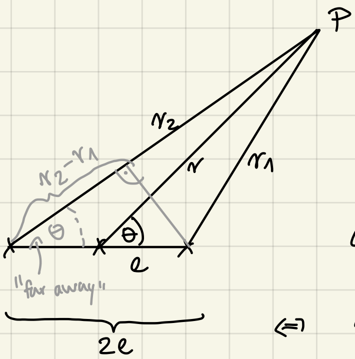
Assumption: Potential of a monopole decays with $\frac{1}{r}$ in anology to the resistivity method (in anoeogy to the gravity method)

$$A = \frac{P}{r_1} - \frac{P}{r_2} = \frac{P(r_2 - r_1)}{r_1 r_2}$$

P: strength of the magnetic field
A: scalar potential field

Far - Field approximation

Rigerous: Taylor exp.



$$\cos(\theta) \approx \frac{r_2 - r_1}{2e}$$

$\Leftarrow \quad r_2 - r_1 \approx 2l\cos(\theta) \quad | \quad$ AND $\quad r_1 r_2 \approx r^2$

$$\boxed{A = \frac{P(r_2 - r_1)}{r_1 r_2} \underset{r \gg l}{\approx} \frac{2pl\cos(\theta)}{r^2}}$$

← orientation angle of P
← distance

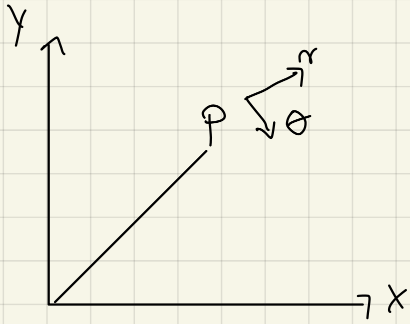
Magnetic dipole field lines

$$\boxed{\begin{aligned}\vec{B} &= -\nabla A = -\nabla\left(\frac{2 p \ell \cos(\theta)}{r^2}\right)\\ &= -\nabla\left(\frac{|\vec{m}|\cos(\theta)}{r^2}\right)\end{aligned}}$$

$|\vec{m}|$: magnetic moment

$$\nabla A = \left(\hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta}\right) A = \begin{bmatrix} B_r \\ B_\theta \end{bmatrix} \begin{matrix}\text{radial component}\\ \text{angular component}\end{matrix}$$

$\hat{r}$: unit vector in r-direction
$\hat{\theta}$: unit vector in θ-direction

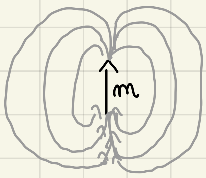


(Exercise)

$$\boxed{\vec{B} = \frac{|m|}{r^3}\begin{pmatrix} 2\cos(\theta) \\ \sin\theta \end{pmatrix}}$$

Important: The magnetic field decays $\sim \frac{1}{r^3}$

Visualization:

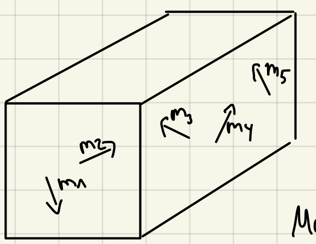


Maxwell eq

Important 2: $\boxed{\nabla \cdot \vec{B} = 0}$!

- field lines are closed
- no sources and sinks
- no magnetic monopoles
- magnetic field is divergent-free

Macroscopic view



$$\vec{M} = \frac{\sum_i \vec{m}_i}{V}$$

Volume magnetisation

Materials can be magnetized in an external field meaning that the individual m_i get orientated along the ext. field lines.

$$\boxed{\vec{M} = \kappa \vec{H}}$$

K: magnetic susceptibility
H: external magnetic field.

$\vec{M}$ in itself is not only a material parameter but it depends on the history of the applied external field $\Rightarrow$ Hysteresis

$$B = \mu_0 (\vec{H} + \vec{M})$$
$$= \mu_0 (\vec{H} + \underbrace{\kappa \vec{H}}_{\text{influence of material}})$$
$$\boxed{B = \mu \mu_0 \vec{H}}$$

H: without material
M: Magnetizations influence