

# Introduction to applied Geophysics R. Drews

# **Gravity Method**

## Problem: What is beneath the surface?





Reference: Public Domain (credit: Mainconfz)

#### Problem: Ice-sheet Mass loss





#### **Reference:**

NASA/Goddard Space Flight Center Scientific Visualization Studio (NASA/GSFC).





#### **Reference:**

Van Camp et al., Geophys. Res. Lett., https://doi.org/10.1002/2016GL070534, 2016



# Evapotranspiration Precipitation Discharge

**Reference:** 

Van Camp et al., Geophys. Res. Lett., https://doi.org/10.1002/2016GL070534, 2016



#### Learning goals today:

Understand how the gravity methods maps spatial and temporal changes in density/mass

Understand the physical background gravitational force, its potential field

Internalize one measurement principle.







Target has a contrast in density.





Target has a contrast in density.

# What is a force?





$$ec{\mathsf{F}} = rac{d}{dt}ec{\mathsf{p}} = \mathsf{m}ec{\mathsf{g}}$$

 $\vec{F} : \text{Force } (N; \text{ kg m s}^{-2}) \\ \vec{p} : \text{Momentum } (N; \text{ kg m s}^{-1}) \\ \vec{g} : \text{Acceleration } (\text{m s}^{-2})$ 

m: Mass (kg)





$$\vec{F} = G \frac{mM}{r^2} \hat{r}$$

$$G = 6.674 \cdot 10^{-11} (m^3 kg^{-1}s^{-2})$$

 $\hat{r}$ : unit vector

r: distance between point masses



## The gravitational constant



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#### **Reference:**

Cavendish, The Proceedings of the National Academy of Sciences , 1798

### The gravitational constant





#### Reference: Westphal et al., Nature, 2021



*G* is the worst known constant in physics:

The gravitational force cannot be shielded.

It averages over "comparatively" large areas.



$$ec{F}=mec{g}$$
  
 $ec{F}=Grac{mM}{r^2}$ í















#### This is a 1D problem.



 $g = G \frac{M}{r^2}$ 





This is a differential equation.







At the Earth's surface  $(R_E)$  g is close to constant and only vertical. (Later we will see that none of this is quite true).















$$r(t)=rac{GM}{2R_e^2}t^2+c_1t+c_2$$

Setting, e.g., c1 = 0 (initial velocity) and  $c_2 = 0$  (initial position) is quite convenient.

By measuring the change of position over time, the mass of the Earth (M) can estimated (given that the radius is known.)

This is the principle of a free-fall gravimeter.



$$r(t) = \frac{GM}{2R_e^2}t^2$$

Go ahead and determine the mass of the Earth M with your Smartphone!

There is an important first-order finding in Earth Sciences that you can (re-) discover. Which one?

#### Vector fields



 $\vec{g} = G \frac{M}{r^2} \hat{r}$ z × l 1 1 k 4 1 

# Potential Field



 $ec{g}=Grac{M}{r^2}\hat{r}$ k . . . . - Conner Л -• -.

.........



#### What is the amount of work required?





$$U(r) = -\int_{\infty}^{r} \vec{g} d\vec{r}$$

$$= -\int_{\infty}^{r} g dr$$

$$= -GM \int_{\infty}^{r} \frac{1}{r^{2}} dr$$

$$= -GM \left[-\frac{1}{r}\right]_{\infty}^{r}$$

$$= GM \frac{1}{r}$$

Potential for a *spherical* mass.



$$ec{g}(r) = -rac{\partial}{\partial r} U(r) = -
abla U(r)$$

It is sometimes easier to calculate the potential of an anomaly and to infer the acceleration via the gradient.

Equipotential lines are *perpendicular* to the field direction.



- ✓ Force, gravitational force/acceleration, gravitational potential
- ✓ Gravity method maps spatiotemporal variability in density/mass
- ✓ Free-fall gravimeters is one measurement principle
- $\times\,$  Spatially distributed structures, sensitivities, data reduction,..

## Problem: What is beneath the surface?





Reference: Public Domain (credit: Mainconfz)





#### Reference:

Jet Propulsion Laboratory, California Institute of Technology, GRACE-FO (a)

#### Problem: Ice-sheet Mass loss





#### **Reference:**

NASA/Goddard Space Flight Center Scientific Visualization Studio (NASA/GSFC).



# You will see the Grace time series in JavaScript enabled PDF viewers such as Acrobat or Okular.

**Reference:** 

NASA, Jet Propulsion Laboratory (https://grace.jpl.nasa.gov/resources/31/antarctic-ice-loss-2002-2020/) accessed 2022.



# Evapotranspiration Precipitation Discharge

**Reference:** 

Van Camp et al., Geophys. Res. Lett., https://doi.org/10.1002/2016GL070534, 2016

# Example: Evapotranspiration





#### **Reference:**

Van Camp et al., Geophys. Res. Lett., https://doi.org/10.1002/2016GL070534, 2016




## Learning goals today:

Understand how different subsurface structures (e.g. caves, sediment infill of valleys, geometry of subduction zones,..) appear in gravity datasets.

Understand the principle of forward and inverse models.

Understand the principle of equivalence.





#### Reference:

Jet Propulsion Laboratory, California Institute of Technology, GRACE-FO (a)



1 
$$Gal = 1$$
 cm  $s^{-2} = 0.01$ m  $s^{-2}$   
1  $mGal = 0.001$  cm  $s^{-2} = 0.00001$ m  $s^{-2}$ 

1 mGal is about 1 millionth of the mean acceleration at the Earth's surface.





#### Reference:

Jet Propulsion Laboratory, California Institute of Technology, GRACE-FO (a)





Xaononl (CC BY-SA 4.0, February 2017)

Newton's shell theorem calculates the gravitational field inside and outside a *spherical shell*.



The field outside a shell is the same as the one from an equivalent point mass

The field inside a shell is zero. Everywhere.

This is useful, e.g., for predicting the gravitational field inside the Earth for different density distributions (i.e. exercises.)



Gravimeters typically only measure the **vertical** component.







Gravimeters typically only measure the **vertical** component.



Profiling across a sub-surface target results in a specific shape of the *vertical* gravity anomaly ( $\rightarrow$  Exercises).





Distance along surface (m)





# 



Surface



For *i* point masses the effect adds up.





$$\vec{g}(\vec{r}) = \sum G \frac{d i v_i}{r_i^2} \hat{r}_i$$

₩

Y ¥











$$ec{g}(ec{r})=G\int
horac{1}{r^2}\hat{r}dV$$



The summation can be replaced by an integration over a volume enclosing a continuous density.



$$ec{g}(ec{r}) = G \int 
ho rac{1}{r^2} \hat{r} dV$$



The integration is a triple integral. Integration limits and coordinates depend on the viewpoint. Example is a Bouger plate, in general not easy to solve ( $\rightarrow$  Exercises).



There are analytical solutions for other shapes (e.g., Nagy 1966 for Prism).





def gravprism(drho, dx1, dx2, dy1, dy2, dz1, dz2):

# gravitational attraction due to "m" prisms at "n" observation point # x1,x2,y1,y2,x1,x2 are coordinates of edges of prisms relative to # observation points. They are m x n matrices.

# Downloaded from: https://www.soest.hawaii.edu/GG/FACULTY/ITD/

# Underlying equations stem from a cartesian integration as detailed in: # Nagy 1966, "The gravitational attraction of a right rectangular prism" # Geophysics VOL. XXX, SO. 2

```
# Calculate distances to all eight corners of the prisms
```

```
R111 = np.sqrt(dx1**2+dy1**2+dz1**2)

R112 = np.sqrt(dx2*2+dy1**2+dz1**2)

R121 = np.sqrt(dx2**2+dy1**2+dz1**2)

R122 = np.sqrt(dx2**2+dy2**2+dz1**2)

R211 = np.sqrt(dx1**2+dy1**2+dz2**2)

R212 = np.sqrt(dx2*2+dy1**2+dz2**2)

R221 = np.sqrt(dx2*2+dy2**2+dz2**2)

R221 = np.sqrt(dx1**2+dy2**2+dz2**2)

R222 = np.sqrt(dx2**2+dy2**2+dz2**2)
```

# Calculate the gravitational acceleration excerted from each corner (?!
g111 = -(dz1\*np.arctan((dx1\*dy1)/(dz1\*R111))-dx1\*np.log(R111+dy1)-dy1\*n;
g112 = +(dz1\*np.arctan((dx2\*dy1)/(dz1\*R112))-dx2\*np.log(R112+dy1)-dy1\*n;
g121 = +(dz1\*np.arctan((dx1\*dy2)/(dz1\*R121))-dx2\*np.log(R121+dy2)-dy2\*n;
g122 = -(dz1\*np.arctan((dx2\*dy2)/(dz1\*R122))-dx2\*np.log(R122+dy2)-dy2\*n;
g122 = -(dz1\*np.arctan((dx2\*dy2)/(dz1\*R122))-dx2\*n;
g122 = -(dz1\*np.arctan((dz1\*dy2)/(dz1\*R122))-dx2\*n;
g122 = -(dz1\*np.arctan((dz1\*dy2)/(dz1\*R12))-dx2\*n;
g122 = -(dz1\*np.arctan((dz1\*dy2)/(dz1\*R12))-dx2\*n;
g122 = -(dz1\*np.arctan((dz1\*dz1\*np.arctan((dz1\*np.arctan(dz1\*np.



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Often deterministic.



Often non-unique (i.e. multiple parameter sets may explain observations). This means *equivalent* options need to be considered.



- ✓ Gravity profiling, gravity anomalies of sub-surface structures
- ✓ Forward modelling (here: Prisms) as a tool for prediction and interpretation.
- ✓ Principle of forward & inverse modelling paired with principle of equivalence.
- $\times\,$  Gravitational field of the Earth, removal of camouflaging effects...



End of video 2



### Learning goals today:

A gravity reading gives you an integrated effect of many processes.

In order to account for a single process (e.g., density variability in sub-surface) corrections are required.



Every gravity survey measures: latitudinal variability, dependency on elevation, the surrounding terrain. excess mass above anomaly, earth & ocean tides. (instr. drift, motion compons.). density variability in the subsurface.



 $\hat{r}$  points radially outwards.  $\hat{\theta}$  corresponds to latitude.



 $\hat{\phi}$  points into or out off the plane. Direction changes with time during the rotation.



The Earth rotates with angular velocity  $\omega = \frac{2\pi}{T}$ .



This results in an inward direction acceleration component perpendicular to the rotation axis.



The Earth rotates with angular velocity  $\omega = \frac{2\pi}{T}$ .



This requires as correction as a function of *latitude*.



Centripedal acceleration at P perpendicular to rotation axis parallel to O'-P (Exercises):

$$g_{r_{\cdot}} = \omega^2 R \cos(\theta)$$

Centripedal acceleration at P perpendicular to rotation axis parallel to O'-P:

$$g_{r.,proj.}=\omega^2R\cos^2( heta)$$

Angular Frequency:  $\omega$ Angular Velocity:  $\vec{v}_r = \vec{\omega} \times \vec{R} \cos(\theta)$ Angular Acceleration:  $\vec{g}_r = \vec{v}_r = \vec{\omega} \times \vec{\omega} \times \vec{R} \cos(\theta)$ 



The latitudinal correction is largest at the equator and zero at the poles.







The reference ellipsoid describes the flattening:  $a_1 = 6357 \text{ km}$   $a_2 = 6378 \text{ km}$  $a_3 = 6371 \text{ km}$ 



Due to the ellipsoidal shape the latitudinal correction is slightly more complicated (but contains no new physics:)

 $g_n = g_e(1 + A\sin^2( heta) - B\sin^2(2 heta))$ 

 $g_e$  statistical reference value at the equator this forms the basis a reference field relative to which anomalies can be define.



Every gravity survey measures: latitudinal variability. dependency on elevation, the surrounding terrain. excess mass above anomaly, earth & ocean tides. (instr. drift, motion compons.). density variability in the subsurface.










The elevation correction references the gravity anomaly to the same datum (e.g., the geoid). How does the gravitational acceleration change with elevation near the Earth's surface?



$$g(r) = G rac{M}{r^2}$$



$$g(r) = g(R_E) + \frac{dg}{dr}|_{R_E}(r - R_E) + \dots$$



$$g(r) pprox G rac{M}{R_E^2} - 2G rac{M}{R_E^3} (r - R_E) + \dots$$



$$g(r) pprox \underbrace{Grac{M}{R_E^2}}_{
m g \ at \ Earth's \ surface} -2Grac{M}{R_E^3}(r-R_E) + ...$$



$$g(r) \approx G \frac{M}{R_E^2} - \underbrace{2G \frac{M}{R_E^3}(r - R_E)}_{\text{change with elevation}} + \dots$$



Taylor expansion near  $r = R_E$ :

$$g(r) pprox G rac{M}{R_E^2} - \underbrace{2G rac{M}{R_E^3}(r-R_E)}_{ ext{change with elevation}} + ...$$

Evaluation at let's say  $r = R_E + 1$  (m) returns a change of  $\delta g(r) \approx -0.3$  mGal per m.



Taylor expansion near  $r = R_E$ :

$$g(r) \approx G \frac{M}{R_E^2} - \underbrace{2G \frac{M}{R_E^3}(r-R_E)}_{-} + \dots$$

 $\delta g(r) \approx -0.3$  mGal per m is large compared to the sensitivity of gravimeters, therefore the gravimeter elevation needs to be determined within centimeters using GNSS.



Every gravity survey measures: latitudinal variability. dependency on elevation. the surrounding terrain. excess mass above anomaly, earth & ocean tides. (instr. drift, motion compons.). density variability in the subsurface.







A neighboring mountain will reduce the measured  $g_z$  independent of target properties.





The terrain correction requires an elevation model and assumptions about the broad-scale sub-surface density. The terrain correction is positive both for surrounding valleys and mountains.



Every gravity survey measures: latitudinal variability. dependency on elevation. the surrounding terrain. excess mass above anomaly, earth & ocean tides. (instr. drift, motion compons.). density variability in the subsurface.























Every gravity survey measures: latitudinal variability. dependency on elevation. the surrounding terrain. excess mass above anomaly. earth & ocean tides. (instr. drift, motion compensation), density variability in the subsurface.



Tides are caused by gravity celestial bodies (i.e. Sun & Moon).

Tidal forces vary across a spatially extended body.

Tidal forces are balanced by centrifugal forces of two (three) body rotations.





# Moon







Gravitational attraction is stronger on the nearside than the farside.





Moon







## Moon

### Earth

Centrifugal force can be projected into radial (i.e. parallel to Earth's gravitation) and parallel component. This leads to the force balance.





Moon The

lunar gravity differential field is responsible for two tidal bulges (i.e. tides twice a day).



Tidal forces vary across a spatially extended body.

Tidal forces are balanced by centrifugal forces of two (three) body rotations.

There is lots of confusion regarding the origin of tides (cf. Matsuda et al. 2015  $\rightarrow$  Ilias).

Tide models or reference measurements can be used for correction

# Tidal signal in gravimetry



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#### **Reference:**

Middlemiss et al., Nature, 2016



Every gravity survey measures: latitudinal variability. dependency on elevation. the surrounding terrain. excess mass above anomaly. earth & ocean tides. (instr. drift, motion compensation), density variability in the subsurface.















Every gravity survey measures: latitudinal variability. dependency on elevation. the surrounding terrain. excess mass above anomaly. earth & ocean tides. (instr. drift, motion compons.), density variability in the subsurface.



 $\delta g_{z,reported} = \delta g_{z,measured} \ - \delta g_{z,lat} \ \pm \delta g_{z,elev} \ \pm \delta g_{z,tides} \ + \delta g_{z,terrain} \ - \delta g_{z,bouguer}$


 $\delta g_{z,reported} = \delta g_{z,measured} \ - \delta g_{z,lat} \ \pm \delta g_{z,elev} \ \pm \delta g_{z,tides} \ + \delta g_{z,terrain} \ - \delta g_{z,bouguer}$ 

free-air:  $\delta g_{z,lat} \pm g_{z,elev}(\pm g_{z,terrain} \pm g_{z,tides})$ bouguer: free-air + bouguer plate (rel. sea level)



- ✓ The gravity method integrates over many processes and data reduction is required.
- ✓ Principle of ellipsoidal flattening & tides.
- ✓ Representation as *free-air* or *bouguer* anomaly maps.
- $\times\,$  Different gravimeter types and applications (e.g. mapping of Geoid).

# END OF VIDEO 3





# Learning goals today:

Gravimeter designs Mapping and definition of the geoid Application examples



Gravimeters are accelerometers free-fall, pendulum, springs satellite orbit perturbations

•••





### Reference: Gneis & Happy SS 2021





Reference: S. Wuestney SS 2021





Reference: J. Noll SS 2023

# Free-fall gravimeters ( $\rightarrow$ Ex.)





Reference: Van Camp et al., EOS, 2017 Credit: Olivier Francis





$$\omega = \sqrt{rac{g}{I}}$$

# Pendulum-based gravimeters ( $\rightarrow$ Ex.)





Reference: M. Roth SS 2023

# Spring-based gravimeters



[cc Reyko, CC-BY-SA3.0]



# Gravimeters: spring-based





Reference: Sandeep CC-BY-SA 3.0]





Reference: NASA GRACE FO / Jet Propulsion Laboratory



# Watch Video 15 Years of GRACE

https:

//www.youtube.com/watch?v=MaxB0vQ2a\_o



Absolute gravimeters are needed

if loop closure if impossible (e.g. intercontinental surveys),

for long-term changes such as isostatic uplift,

as basestations for relative surveys.

Relative surveys are always easier to conduct and loop closure can cancel many error sources (e.g., instrument drift).



Top *absolute* gravimeters  $\sim 1\mu$ Gal ( $10^{-9}g$ ) Top *relative* gravimeters  $\sim 10\mu$ Gal ( $10^{-8}g$ ) Typically only  $g_z$  is measured.

## Application examples: Geoid





#### **Reference:**

Result from GRACE-FO (Credit: GFZ Potsdam)





Geoid is a real-world equipotential line approximating sea level.

It is referenced to the geometric ellipsoid.





The reference of elevation is a constant source of confusion.

The geoid defines the local vertical direction.





### Upwarping of geoid indicates mass excess.

Downwarping of geoid indicates mass deficit.

# Application examples: Andes





# Application Examples: Andes



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- (a) Topography.
- (b) Measured  $g_z$ .
- (c) Reference gravitational field.

### **Reference:**

From Clauser 2018 (after Schmidt and Götze 2006 in Hackney 2011)

# Application Examples: Andes



'IRINIGEN

- (a) Free-air reduction
- (b) Bouguer reduction
- (c) Free-air + Bouguer reduction

#### **Reference:**

From Clauser 2018 (after Schmidt and Götze 2006 in Hackney 2011)

### Application Examples: Andes



### (a) Free-air anomaly

- (b) Bouguer anomaly
- (c) Isostatic anomaly (assuming 20 km root depth)

#### **Reference:**

From Clauser 2018 (after Schmidt and Götze 2006 in Hackney 2011)



In mountains the elevation correction dominates

A negative Bougues anomaly is indicative or mountain roots

Isostatic anomaly is not hypothesis free, but markes the Andes as an active orogen which is not yet in isostatic equilibrium

Bouguer anomaly map of Germany will be treated in exercises.



# Take aways from this video:

There are different types of gravimeters (absolute vs. relative).

The geoid is an important equipotential line.

Bouguer and free-air anomaly maps offer important insights into sub-surface structures.

# END OF VIDEO 4

