scalar ⟺ number: Mass; Density; Speed — number

vectors ⟺ magnitude / direction: Velocity; Force — vector

tensors ⟺ magnitude / directiction / place where things happen: Stress — tenso

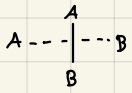
Derivatives take different forms in different coordinate systems — $\vec{\nabla}$ hus different shapes
> cartesian > spherical > cylindical

## VL 16.05.23

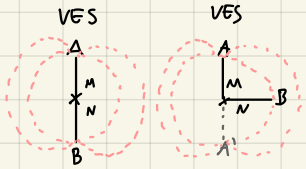
VES: perfectly suited to map vertical variability (~ analog to refraction seismic without dip)
↖ vertical electrical sounding

How would we notice lateral variability (or lack therefore)?

1) Suggestion: more data e.g.




2) Use half Schlumberger arrays

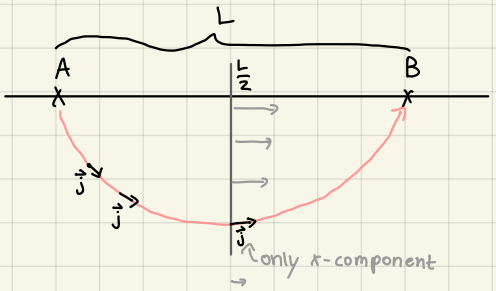



←switch between A and A' and that is only a cable switch
"almost equal geometry"

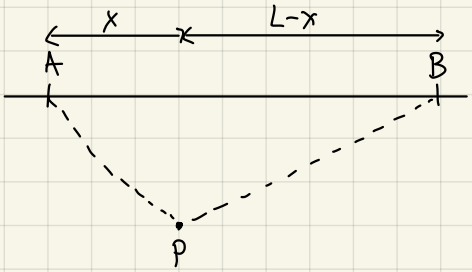
If horizontal stratification then same readings for A and A'
If lateral variability those readings will be different

## Depth of investigation (DOI)

DOI: essentially defines a depth interval from which most (qualitative) of the signal comes.




$j_x(z) = ?$




Potential

$$U = \frac{I}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{I}{2\pi}\left(\underbrace{\frac{1}{\sqrt{x^2+y^2+z^2}}}_{(x^2+y^2+z^2)^{-\frac{1}{2}}} - \frac{1}{\sqrt{(L-x)^2+y^2+z^2}}\right)$$

$$\vec{j} = \sigma\vec{E} = -\sigma\nabla U$$

$$j_x = \frac{\partial}{\partial x}U = -\sigma\,\frac{I}{2\pi}\left(-\frac{1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}}\cdot 2x - \left(+\frac{1}{2}((L-x)^2+y^2+z^2)^{-\frac{3}{2}}\cdot 2(L-x)\right)\right)$$

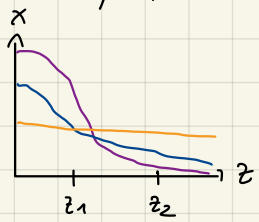
$$= -\sigma\left[-\frac{I}{2\pi}\left((x^2+y^2+z^2)^{-\frac{3}{2}}\cdot x + ((L-x)^2+y^2+z^2)^{-\frac{3}{2}}\cdot(L-x)\right)\right)$$

$$= \delta \frac{I}{2\pi} \left( \frac{x}{(x^2+y^2+z^2)^{\frac{3}{2}}} \cdot x + \frac{L-x}{((L-x)^2+y^2+z^2)^{\frac{3}{2}}} \right)$$

Now: evaluate at center plane: $x = \frac{L}{2}$

$$j_x\left(x=\frac{L}{2}\right) \sim \frac{L}{\left(\frac{L^2}{4}+y^2+z^2\right)^{\frac{3}{2}}}$$

Center line: ($y=0$)




L=2
L=3
L=4 → more current at deeper depth
→ current distributes over larger area

Larger L → increased depth of investigation

One way of defining DOI: Integrate the total contribution for a given depth interval

$$DOI \sim \int_{z_1}^{z_2} dz \int_{-\infty}^{\infty} dy \; \frac{L}{(z^2+y^2+\frac{L^2}{4})}$$

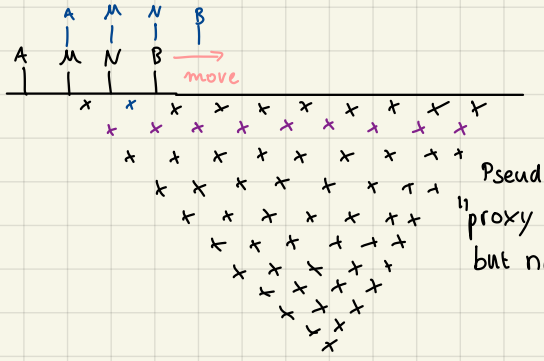
"doable" but "technical"

Some people chose $z_1 = 0$ (with $z_2 = \infty \rightarrow DOI = I$) others don't hence there are different values in books.
In all cases DOI depends on array type.
In all cases $DOI \ll L$ Wenner array: $DOI \sim 0{,}12$ ("broadly speaking"). The truth is in the forward model.
⇒ resistivity mapping in that sense is often restricted to <20m

## Pseudo-2D-Profiling

1) Wenner array $\overline{AM} = \overline{MN} = \overline{NB} = a$
→ fix a
2) get readings while moving (put $s_a \sim 0{,}1\,a$)
3) reach end an increase a

repeat




Pseudo-2D-cross-section
"proxy for true resistivity but not the real one"

## Self-potential

Self potential method measures potential differences from natural sources (passive technique).
Fun fact: easiest method in the field.
(interpretation tricky)


Voltmeter and electrodes

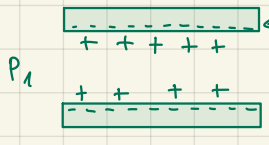
Natural sources:
- streaming potentials (groundwater, seepage of dykes)
- redox potentials (ore expeorations)
- thermo-potentials (conductivity function of temp.) (geothermal reservoirs)
- membrane-potentials (stratigraphy), also contact potential

### redox potentials

e- → →
anaerobic zones ⊖ — aerobic zones ⊕

→ chemically driven

### streaming potentials

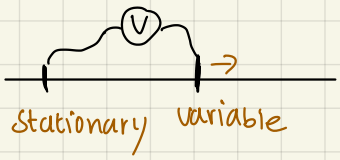



- +Ions transported → current → E-Field → Potential-Field
- electric double layer

membrane potentials
- electrodes in borehole → contact potentials
- materials different acceptence for e.g. ⊕ charged ions

→ potential differences (no current flow)

Self-potential Measurement




↯ large contact resistence between sticks and subsurface
→ dominates

→ need non polarizable electrodes