Eberhard Karls Universität Tübingen

Introduction to applied Geophysics

R. Drews

Magnetic Method

Learning Goals

Learning goals in the next steps

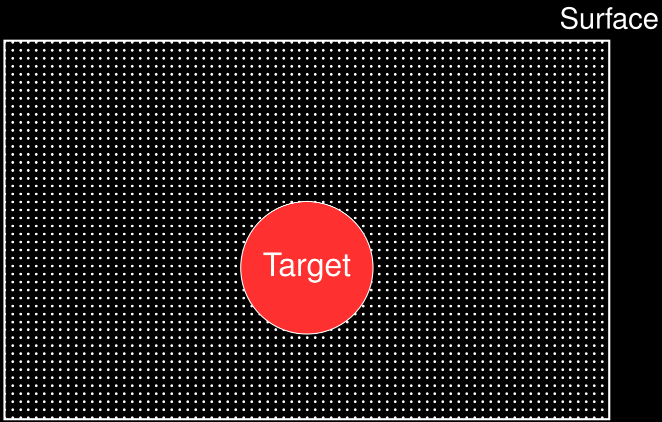
- Examples: How can we use magnetism as process for geophysical prospecting?
- Theory: Derive an expression for a magnetic field in analogy to gravity method.
- Take away: Magnetic anomalies will be significantly more complicated than gravity anomalies.

Suggested Literature

Literature (there is a lot)

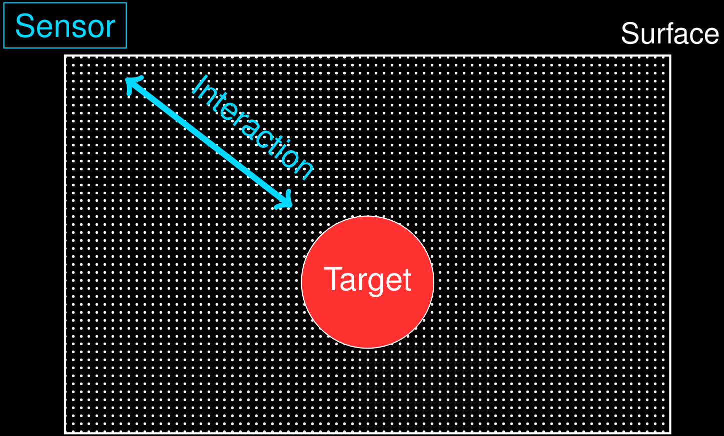
- Einführung in die Geophysik (Clauser, Chapter 5)
- Gravity and Magnetics - Principles, Practices, and Applications

Mapping differences in magnetic properties



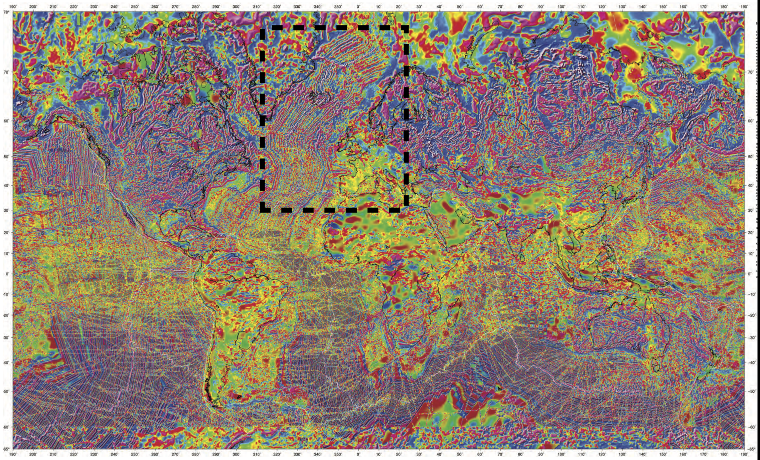
Target has a contrast in magnetic properties.

Mapping differences in magnetic properties



Target has a contrast in magnetic properties.

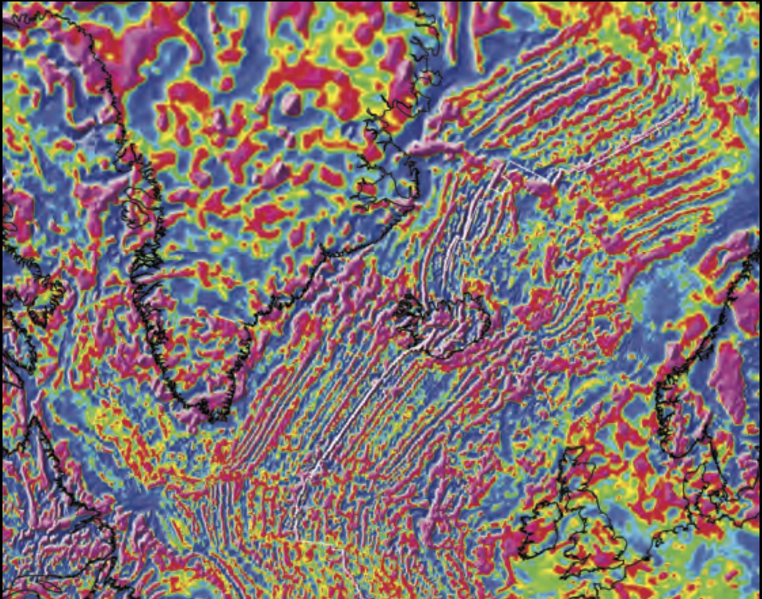
Example Applications: Global Scale



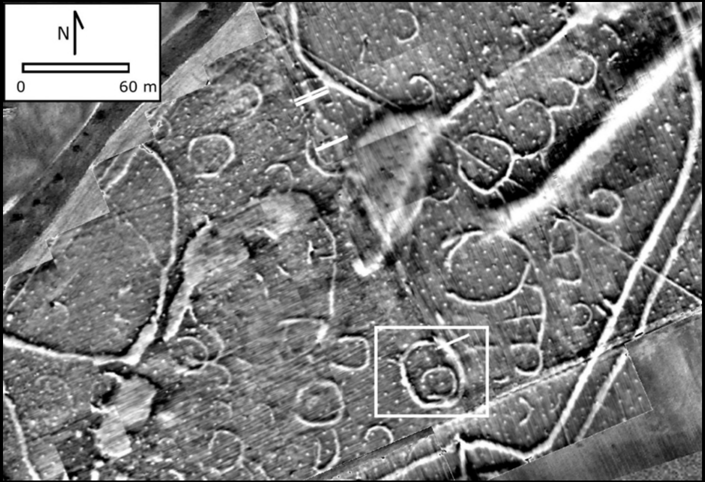
Units nT; Reference:

Korhonen et al., 2007, C. Geolog. Map., Paris, France.

Example Applications: Global Scale

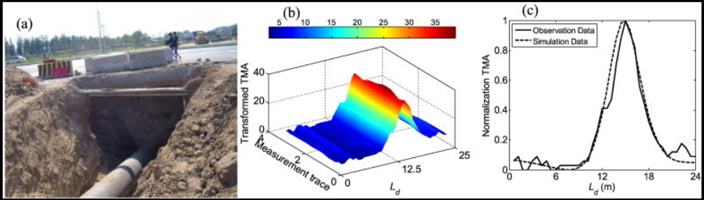


Example Applications: Regional Scale



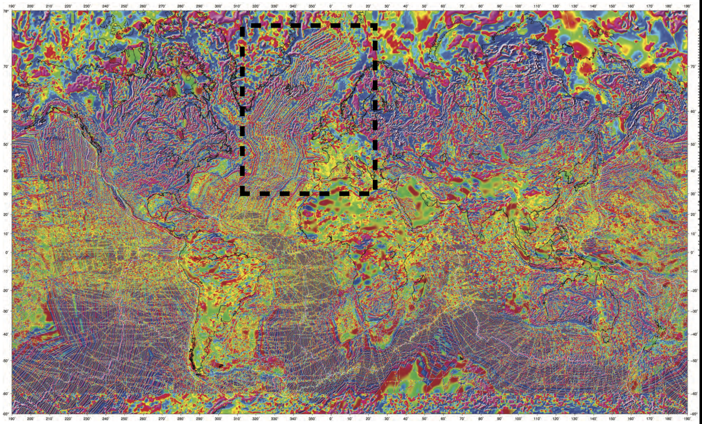
[Ciminale et al., 2009, Archeosciences.]

Example Applications: Local Scale



Units nT; Reference:

Guo et al., 2014, J. Appl. Geophys.



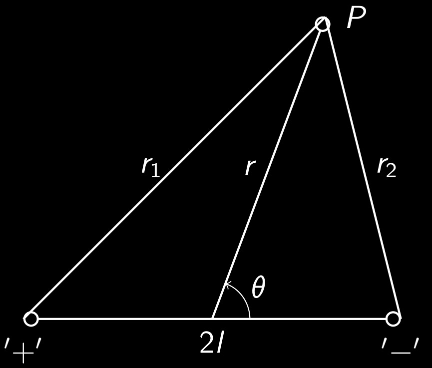
[Korhonen et al., 2007, C. Geolog. Map., Paris, France.]

What is the shape of magnetic fields and how can we describe it mathematically?

The shape of a magnetic field:

- Let's use an analog to the point mass in the gravity method
- Let's represent 'north' and 'south' poles with point charges (there are no monopoles!)
- Let's derive the potential A of a pair of positive-negative point masses
- Let's get the field using $\vec{B} = \nabla A$

The magnetic dipole potential



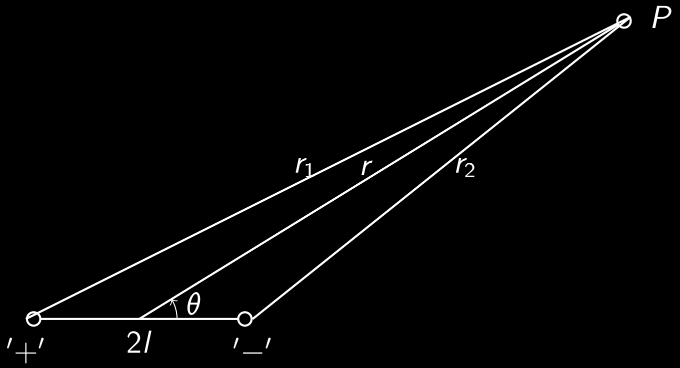
$\frac{p}{r_1}$ $\qquad$ $-\frac{p}{r_2}$

Use 'magnetic' potential in analogy to gravity potential

The magnetic dipole potential

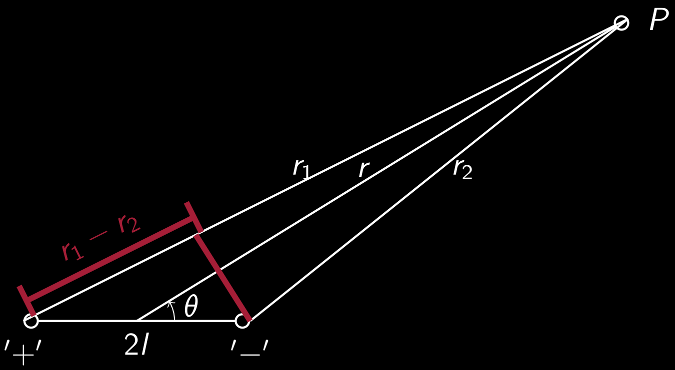
$$
\begin{aligned}
A &= \frac{p}{r_1} - \frac{p}{r_2} \\
&= \frac{p(r_2 - r_1)}{r_1 r_2}
\end{aligned}
$$

The magnetic dipole potential: farfield



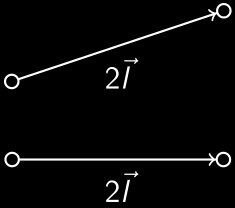
$$A = \frac{p(r_2 - r_1)}{r_1 r_2} \qquad \text{for} \qquad r \gg l$$

The magnetic dipole potential: farfield



$$A = \frac{p(r_2 - r_1)}{r_1 r_2} \approx \frac{2lp\cos(\theta)}{r^2} \qquad \text{for} \qquad r \gg l$$

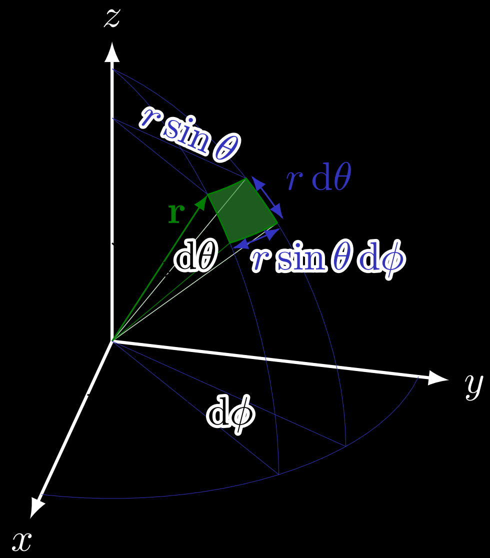
The magnetic dipole potential: farfield



$\vec{m} = 2\vec{l}p$ (dipole moment [A m^2])

$$
\begin{aligned}
A &= \frac{p(r_2 - r_1)}{r_1 r_2} \\
&\approx \frac{2lp\cos(\theta)}{r^2} \\
&= \frac{|\vec{m}|\cos(\theta)}{r^2}
\end{aligned}
$$

Mathematical technicalities



Mathematical technicalities

Unlike cartesian coordinates, the unit vectors in polar coordinates ($\hat{\theta}$, $\hat{\phi}$ not $\hat{r}$) are functions of positions. Therefore the derivatives take a different form in different coordinate systems.

$$\nabla A = (\hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta})A = \begin{bmatrix} B_r \\ B_\theta \end{bmatrix}$$

$$\nabla \cdot \vec{B} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 B_r\right) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta) B_\theta\right)$$

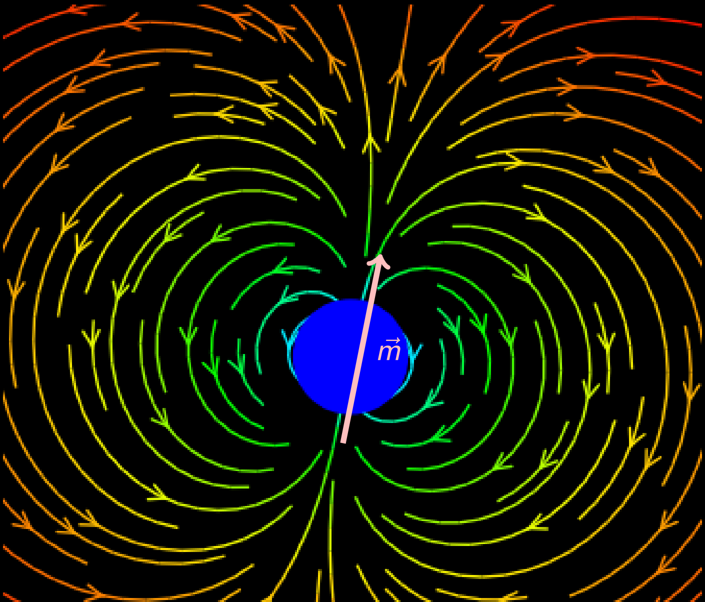
No need to be scared. Totally doable with some exercises.

In analogy to the gravity potential:

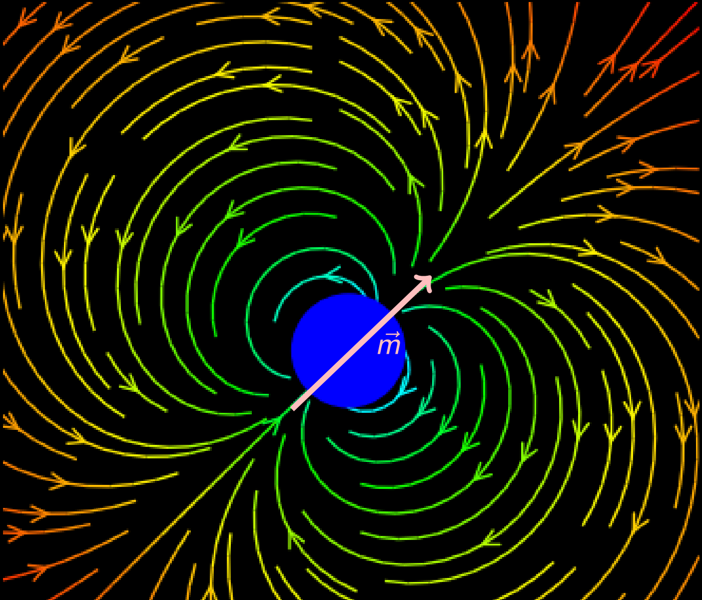
$$\vec{B} = -\nabla A$$
$$\approx -\nabla \frac{|\vec{m}| \cos(\theta)}{r^2}$$
$$= \frac{|m|}{r^3} \left(2\cos(\theta)\hat{r} + \sin(\theta)\hat{\theta}\right)$$

(Derivation in Exercises.)

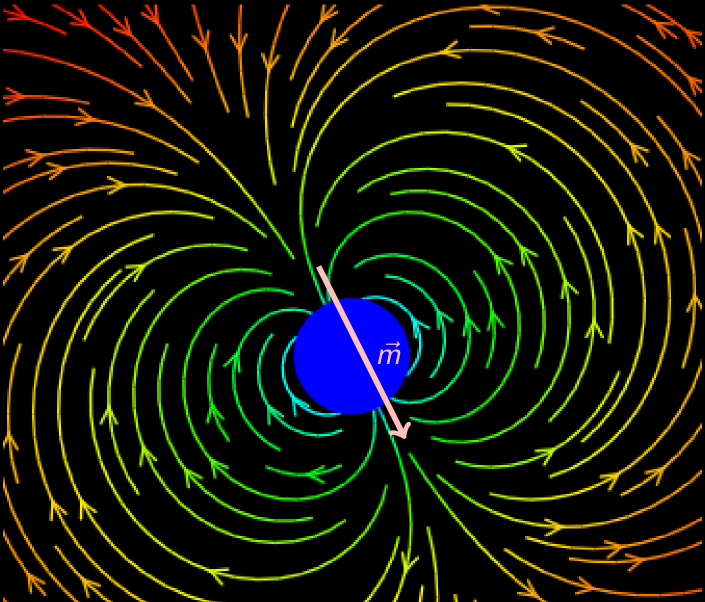
The dipole field visualized



The dipole field visualized



The dipole field visualized

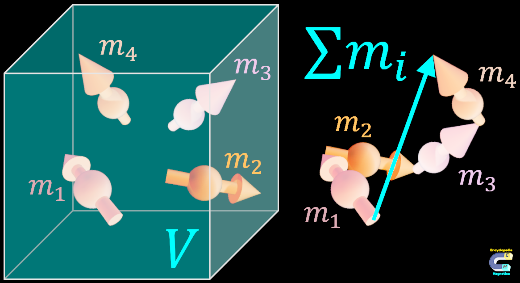


- The dipole approach is simplest option because there are no magnetic monopoles.
- The dipole field has closed field lines and is oriented with the dipole moment $\vec{m}$.
- The dipole field is the basic 'unit' in magnetics.

Dipole summary 1

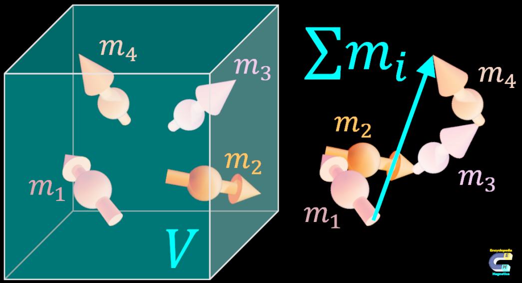
- The dipole field is always divergence free ($\nabla \cdot \vec{B} = 0$) (Exercises.)
- The dipole field as derived here has a microscopic point of view (cf. point mass)
- Unlike the gravitational force ($\sim r^{-2}$) the dipole field decays with $\sim r^{-3}$ because the opposite charges have a partially canceling effect.

Macroscopic Property: Magnetization M



Units nT; Reference:

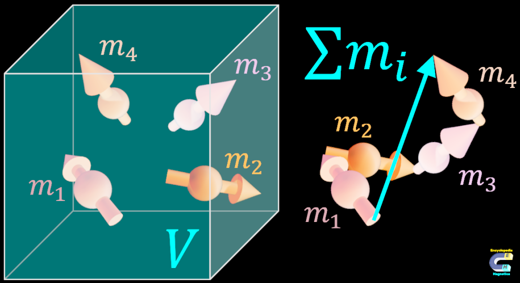
$$\vec{M} = \frac{\sum_i \vec{m}_i}{V}$$



Units nT; Reference:

- The *volume Magnetisation* $\vec{M}$ [A m^{-1}] describes the macroscopic magnetic properties of an object.

Macroscopic Property: Magnetization M

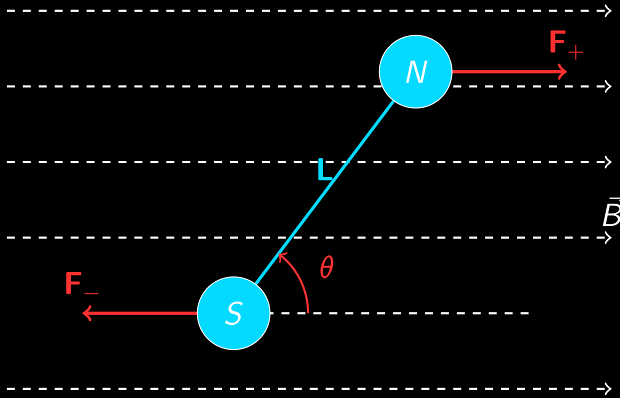


Units nT; Reference:

S. Zurek, Encyclopedia Magnetica, CC-BY-4.0

- The *volume Magnetisation* is is tightly linked to the magnetic polarisation in analogy to an electric polarisation.

External fields can change direction of magnetisation



Net torque:

$$\vec{\tau} = \vec{M} \times \vec{B}$$

- An external field can *magnetize* material by ordering their individual dipole moments (from $\vec{M} = 0$ to $\vec{M} \neq 0$).
- This is no equivalent in the gravity method.
- The degree of magnetization is material dependent and described with the *magnetic susceptibility* k

$$\vec{M} = k\vec{H}$$

- $\vec{H}$ is termed the *magnetizing field* or *the magnetic field strength*.
- k can be strong, weak, positive or negative which defines the material as dia-, para, ferro- magnetic (discussed later.)
- What is the source of the magnetizing field?

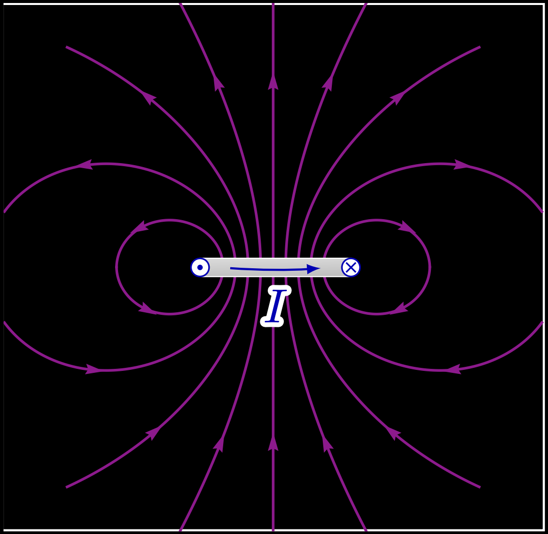
$$\vec{M} = k\vec{H}$$

Newton in mechanics is Maxwell in electromagnetics:

$$\nabla \times H = \vec{j} + ..$$

$\vec{j}$ is the electrical current density flowing along a path (wire). The magnetizing field can take very complicated forms as a function of the wire geometry (cf. Bio-Savart law), but there is are least one simple solution.

Magnetic dipole field of a current loop



Dipole summary

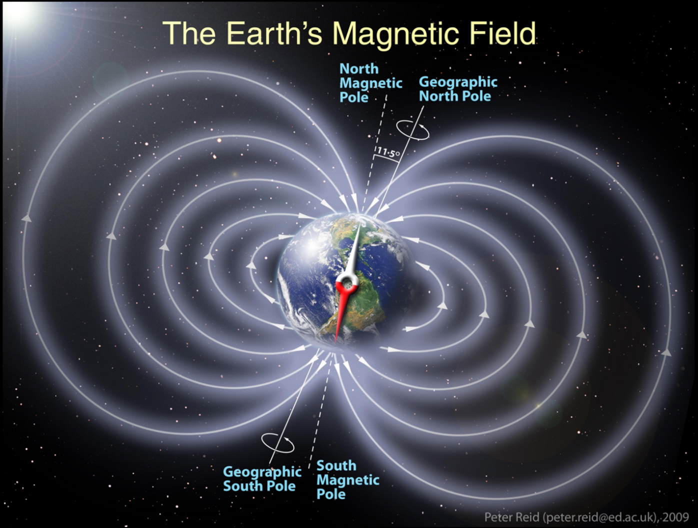
- A closed loop with a current is the source of a dipole field (and the calculation with positive/magnetic monopoles is a trick that makes it easier). Alternatively use Bio-Savart or a multipole expansion.)
- The origin of magnetism are electrical currents, both on a microscopic (e.g. electron around nucleus) and a macroscopic scale (e.g. antenna).

- Describes the total field including the volume magnetization. It is termed *magnetic induction*, or *magnetic flux density*.

$$\begin{aligned}\vec{B} &= \mu_0\left(\vec{H}+\vec{M}\right)\\ &= \mu_0\left(\vec{H}+k\vec{H}\right)\\ &= \mu\mu_0\left(\vec{H}\right)\end{aligned}$$

μ_0 magnetic permeability of vacuum [N A^{-2}]. The mag. induction is measured in Tesla [T].

Magnetic field of the Earth



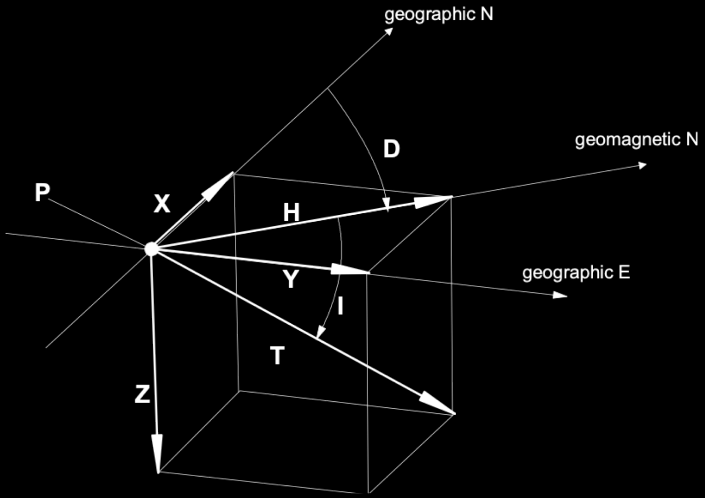
[P. Reid, Univ. Edinburgh, UK]

- Earth’s magnetic field is to a good approximation a dipole field (in the absence of near-surface anomalies).
- The origin of this field is deep in Earth’s interior (conduction currents in outer mantel).
- The dipole moment is not parallel to rotation axis but shifts. Hence there is an offset between magnetic and geographic poles.
- The Earth’s magnetic field is variable on contemporary and geologic timescales (Exercises)

Magnetic field of the Earth

- North seeking end of magnetic needles dips downwards in northern hemisphere.
- South seeking end of magnetic needles dips downwards in southern hemisphere.

Magnetic field of the Earth



Parameters of the Earth magnetic field at point P

Magnetic field of the Earth

T: total field ($T^2 = H^2 + Z^2 = X^2 + Y^2 + Z^2$)
Z: vertical component of T
X: component of T in geographic N-S direction
Y: component of T in geographic W-E direction
H: horizontal component of T
I: inclination (angle versus horizontal)
D: declination (angle versus geographic N)

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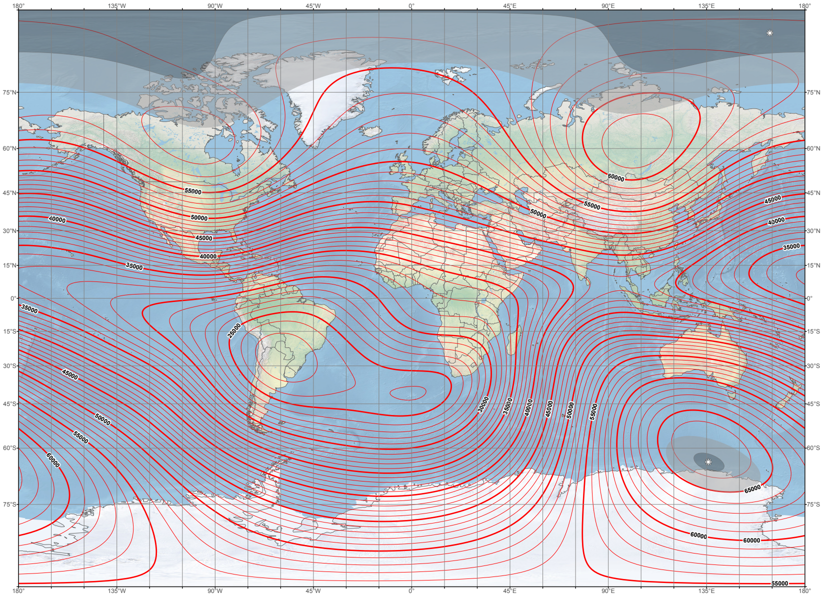
US/UK World Magnetic Model - Epoch 2020.0
Main Field Total Intensity (F)



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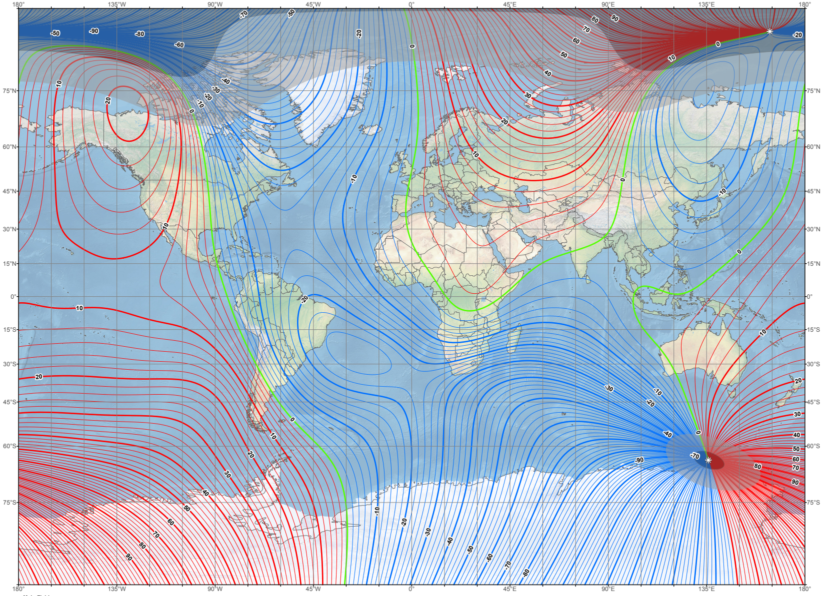
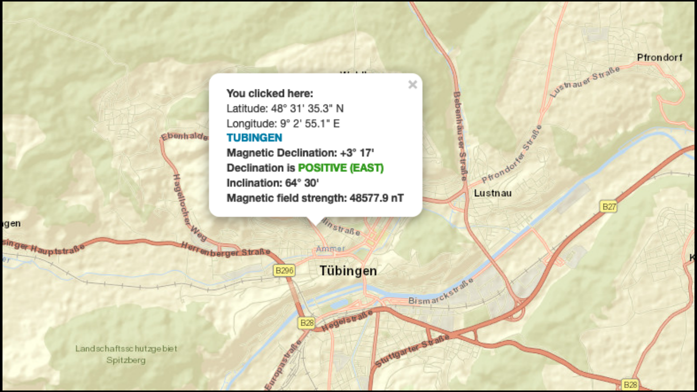
US/UK World Magnetic Model - Epoch 2020.0
Main Field Declination (D)

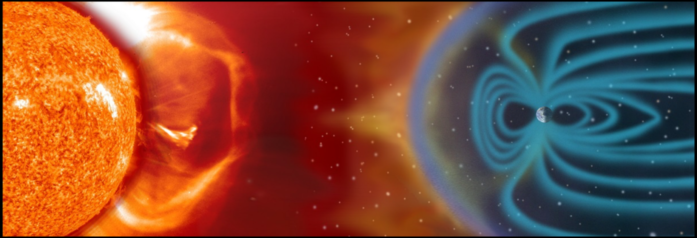


Magnetic field of the Earth



[Tübingen $T \sim 49000nT$]

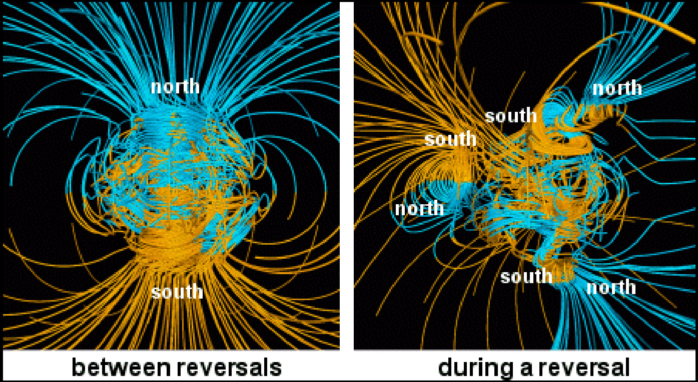
Magnetic field of the Earth



[NASA]

- The Earth's magnetic field is time variable, on contemporary timescales because of solar storms. Space weather is a real thing and can be detected, e.g., with GPS measurements. A reference station in magnetics is more important than in the gravity method.

Magnetic field of the Earth



[Model of Glatzmeier & Roberts]

- Magnetic pole reversal and shifts on longer timescales. Great technique for dating!

Repetition Word Cloud

Magnetic Dipoles, Magnetic Potential (A), Farfield Approximation, magnetic induction (B), magnetic field strength (H), magentic dipole moment, volume magnetization, magnetic susceptibility, induced magnetization, currents as sources (i.e loop results in dipole field), the Earth's magnetic field, Geodynamo, Declination (D), Inclination (I), Total Field (T), Time Variability (solar winds, core)

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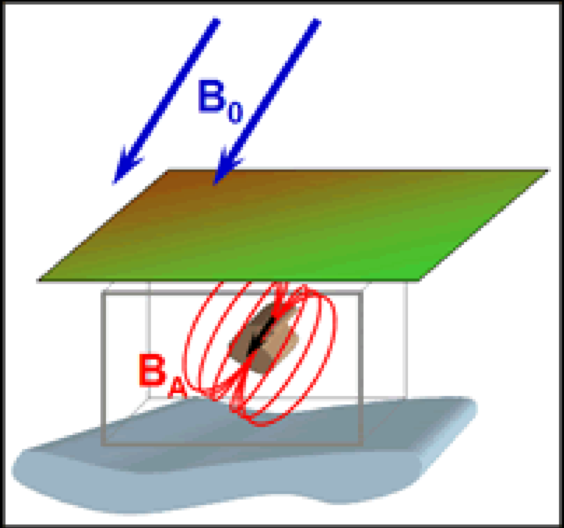
Learning goals:

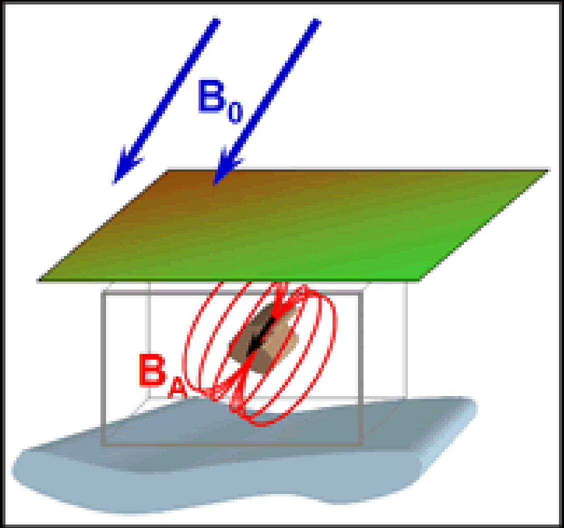
- Understand that the shape of magnetic anomalies depends on the Earth's magnetic field which varies spatially.
- Understand why anomalies often have dipole nature.
- Understand the restriction that those derivations only work for induced and not remanent magnetization.

Principals of magnetic surveys



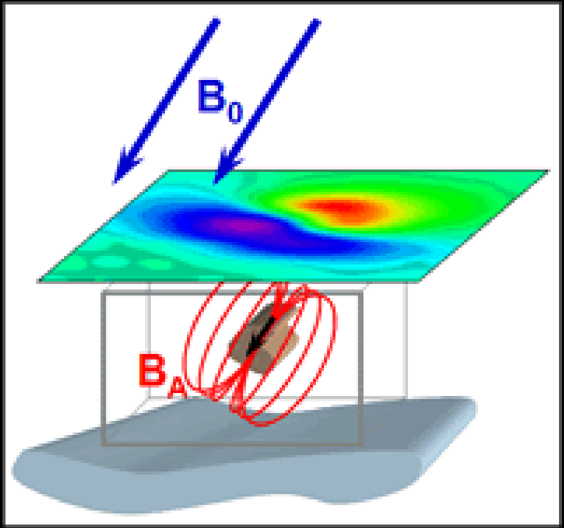
[2017, GeoSci Developers.]

Principals of magnetic surveys



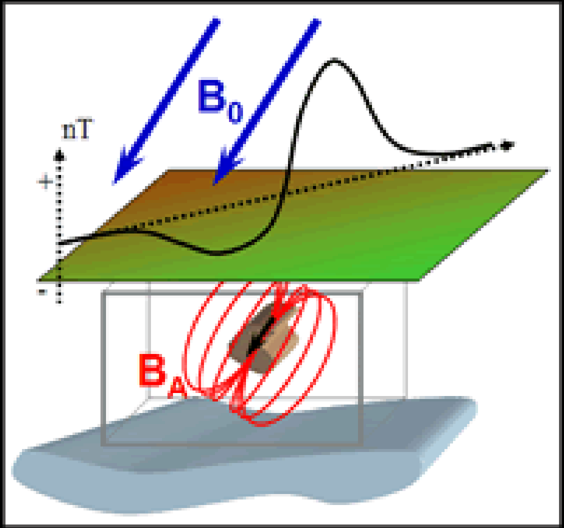
[2017, GeoSci Developers.]

Principals of magnetic surveys



[2017, GeoSci Developers.]

Principals of magnetic surveys



[2017, GeoSci Developers.]

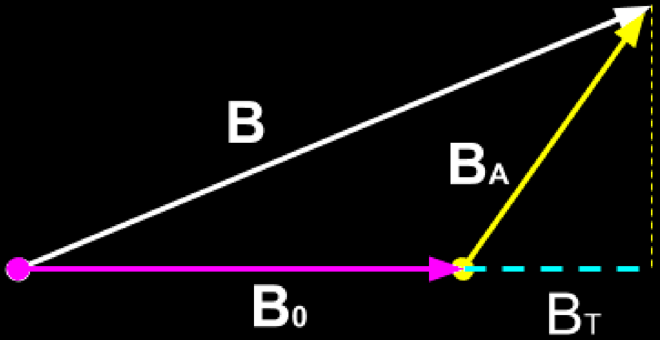
Strategy for expectations of induced magentic dipoles

Induced magnetic dipole:=Direction of $\vec{M} \parallel \vec{B}_0$

- Draw S-End, N-End and inclination of planned profile
- Draw $\vec{m}$ and corresponding dipole field at depth
- Analyse superposition of B_0 and dipolfield.
- (Assume that the anomaly is essentially parallel to B_0)

Approximation for the total field anomaly

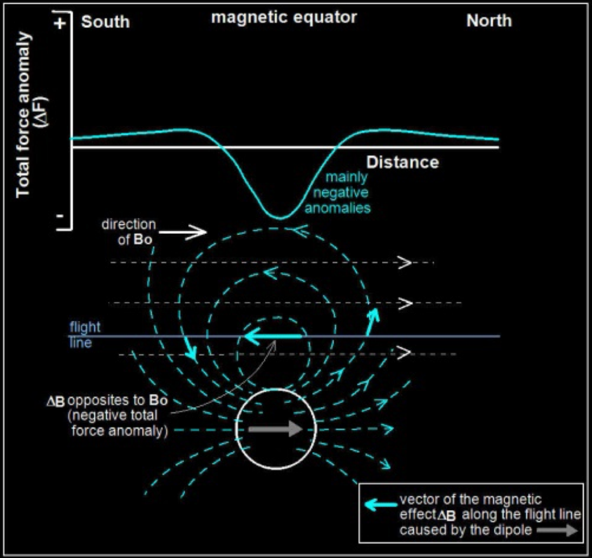
Induced magnetic dipole: Direction of $\vec{M} \parallel \vec{B}_0$



$$B_T = |\vec{B} \cdot \vec{\hat{B}}_0|$$

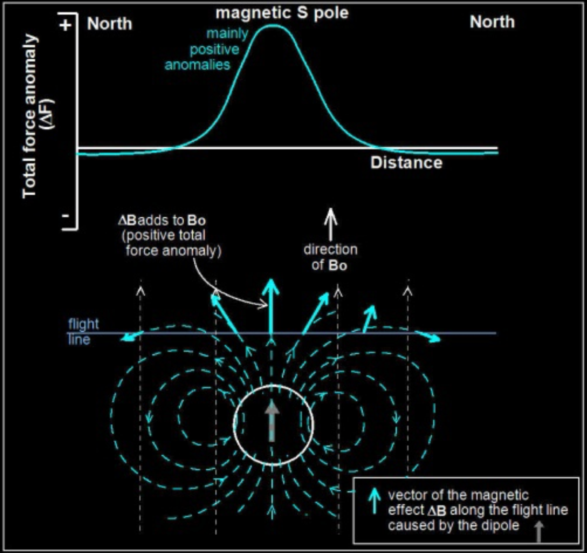
This works well for $|B_0| >> |B_A|$

Induced magnetization examples



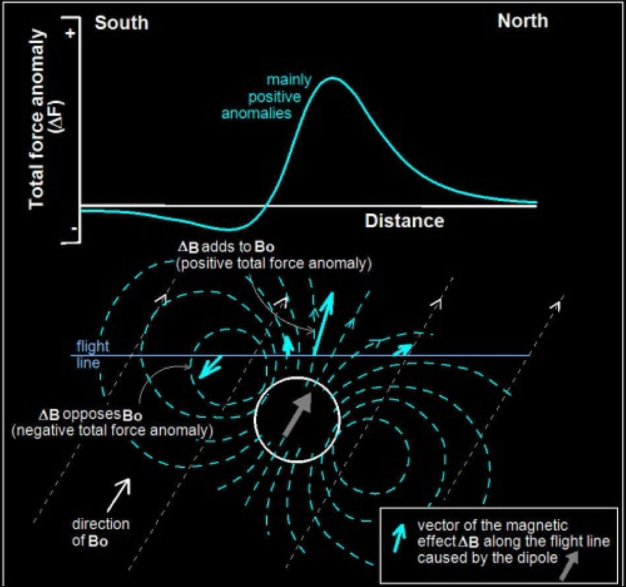
(d) Magnetic equator

Induced magnetization examples

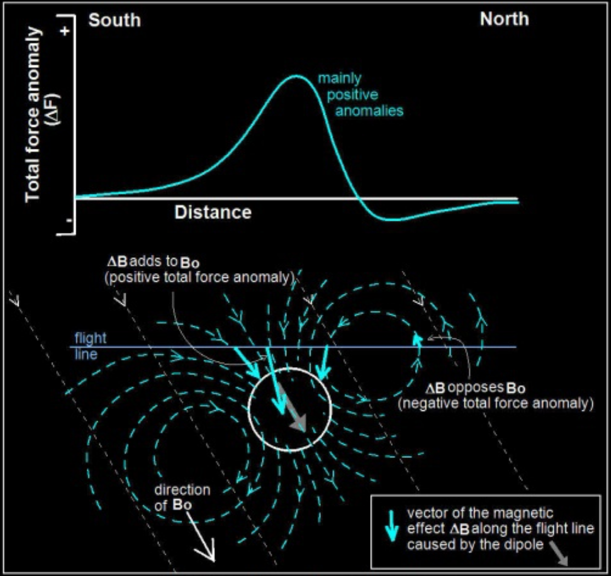


(c) Magnetic south pole

Induced magnetization examples



Induced magnetization examples



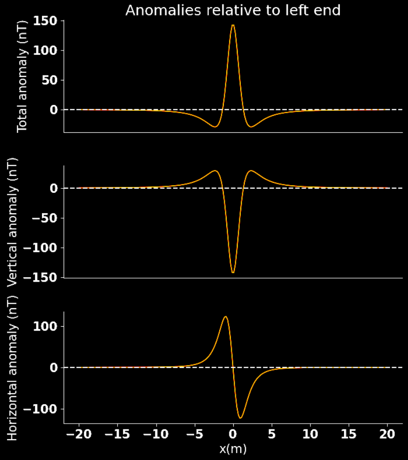
Interpreting magnetic anomalies:

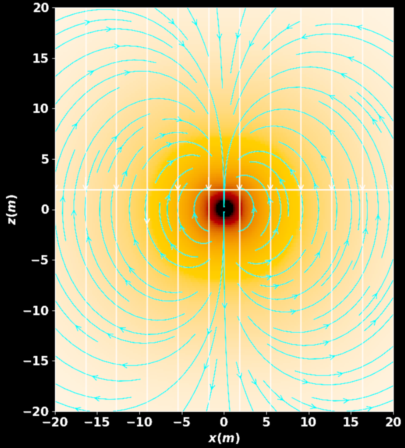
- Unlike in the gravity method, magnetics often measures in 3D
- Interpretation of ΔT, ΔH, ΔZ, ...
- Commonly also the vertical gradient of ΔZ (cf. exercises)

Demonstration of forward model (Exercises)

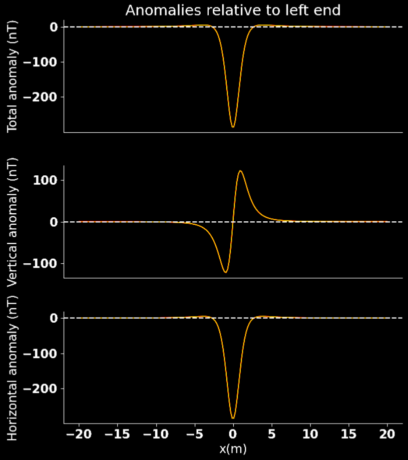

```
## ----------------------------
## Let's do the standard imports
import sys
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
import matplotlib.colors as colors
## This is a function, returning the value of a H field
## at r and theta from the origin
## where the magnetic dipole is located.
# Assume volume Magnetization M [A/m] parallel to external field.
def B(r, theta,M):
    """Return t,he magnetic field vector at (r, theta)."""

    fac = (M / r)**3
    return 2 * fac * np.cos(theta + alpha), fac * np.sin(theta + alph

#Background field strength in nTesla somewhat comparable to Tuebingen
Bfm = 40000;
# Induced Magnetization in A/m*mu_0; this is target material and geome
M =  10
# Deviation of magnetic pole from vertical axis,
alpha = np.radians(-90)

 #....
```

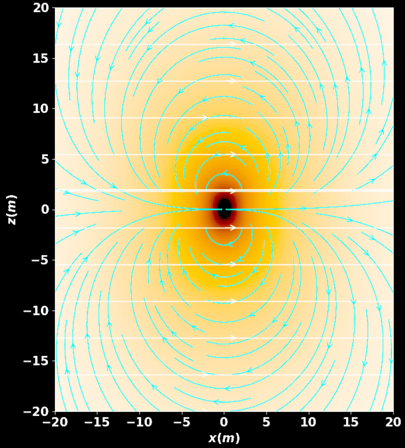
Demonstration of forward model (Exercises)



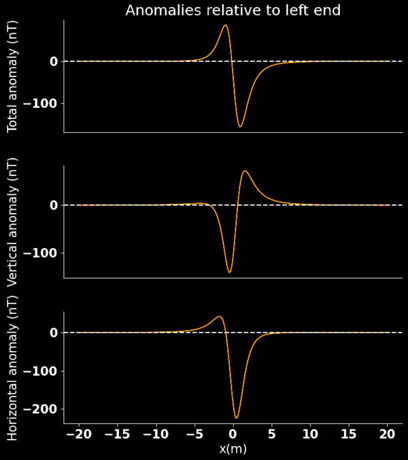


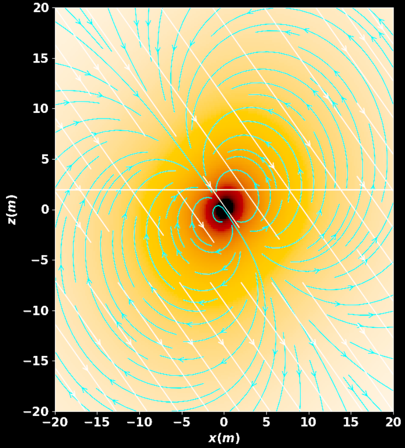
Demonstration of forward model





Demonstration of forward model (Exercises)





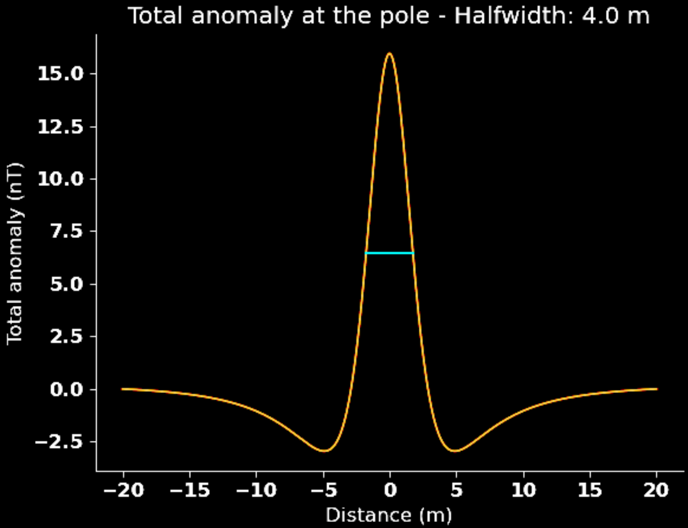
Half-width rules

Similar to the anomaly in gravity surveys, we can also apply a half-width rule in magnetics to estimate the depth of the object.

$$d \sim HW$$

Check this with the forward model. (This is only a rule of thumb).

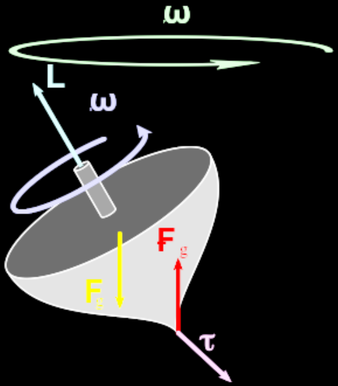
Demonstration of forward model (Exercises)



Induced magnetization examples

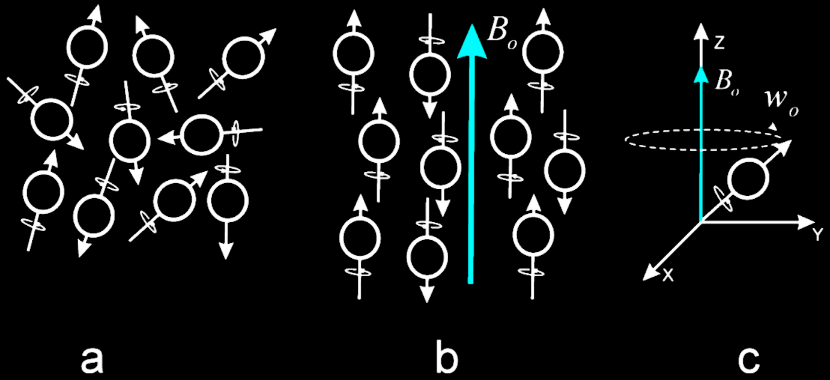
- Unlike for the case of gravity, magnetometers may measure all directions of the anoamlie.
- Total field: Proton Precession (cf. Overhauser)
- Components: Fluxgate (cf. Förster)
- Vertical Gradients (Gradiometer)

Types of Magnetometers: Proton Precession



[XSnelgrove (CC-BY-SA 2.5)]

Types of Magnetometers: Proton Precession



[F. Ordonez 2014]

At room temperatures spins are not aligned as Earth's magnetic field is too weak to do so.

Types of Magnetometers: Proton Precession

- Protons have angular momentum and magnetic moment.
- External magnetic field (from coil) induces torque, so that mini magnetic moment tends to aligns parallel to the external field.

Types of Magnetometers: Proton Precession

- Hydrogen atoms have angular momentum and magnetic moment.
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- After alignment, external field is switched off so that Lamor frequency now responds to Earth magnetic field.

$$\omega = \gamma_p |B|$$

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- Idea: Measure ω and infer $|B|$

$$\omega = \gamma_p |B|$$

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Practical implementation:

- Use liquid with many hydrogen atoms (e.g. water or kerosene).
- Align their magnetic moments with strong, external magnetic field.

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- Use liquid with many hydrogen atoms (e.g. water or kerosene).
- Align their magnetic moments with strong, external magnetic field.
- Measure the precession frequency (how?)

Induction: Time variable B field induce currents

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

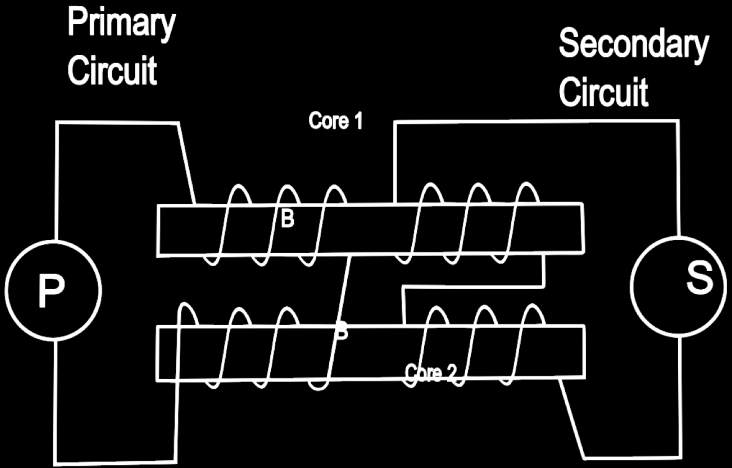
- The time variable magnetic fields from the relaxation induce currents in the surrounding coil which can be measured.

Types of Magnetometers: Proton Precession

Types of Magnetometers: Proton Precession

- Advantage: Positioning not so important
- Advantage: Sensitivity < 0.1 nT
- Disadvantage: Base station required (sometimes).
- Disadvantage: Measurement time > 1s
- Related techniques: MRI imaging in medicine

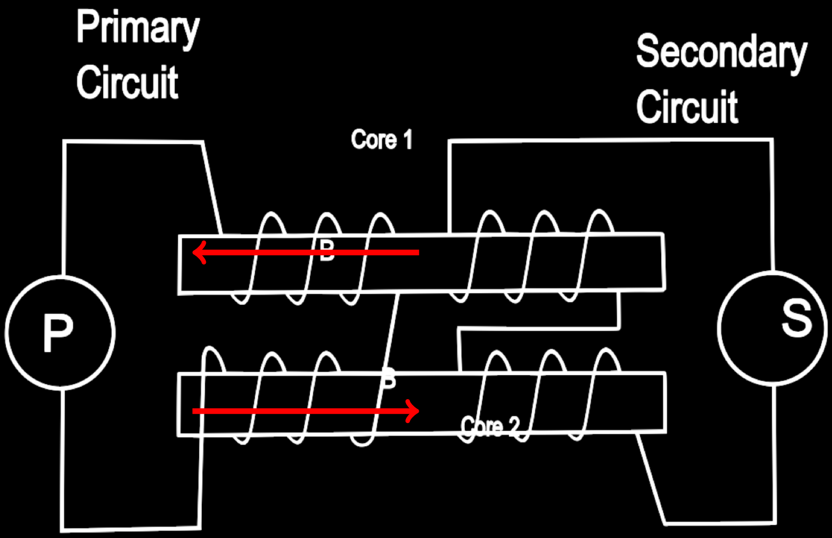
Types of Magnetometers: Fluxgate



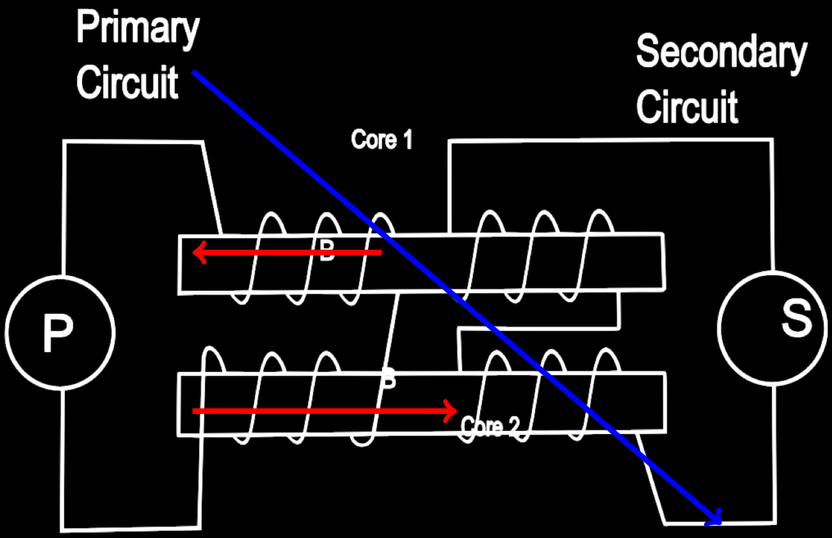
Fluxgate magnetometer

- Induce low-frequencey (kHz) current in primary circuit.
- The magnetic fields in secondary coil will be balanced in absence of external field.
- External field causes imbalance between primary and secondary coils that can be used to measure B_x, B_y, B_z depending on coil orientation.

Types of Magnetometers: Fluxgate



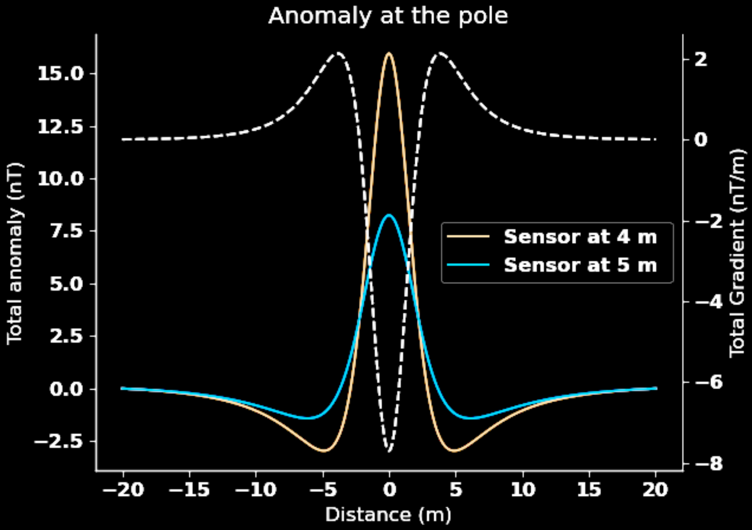
Types of Magnetometers: Fluxgate



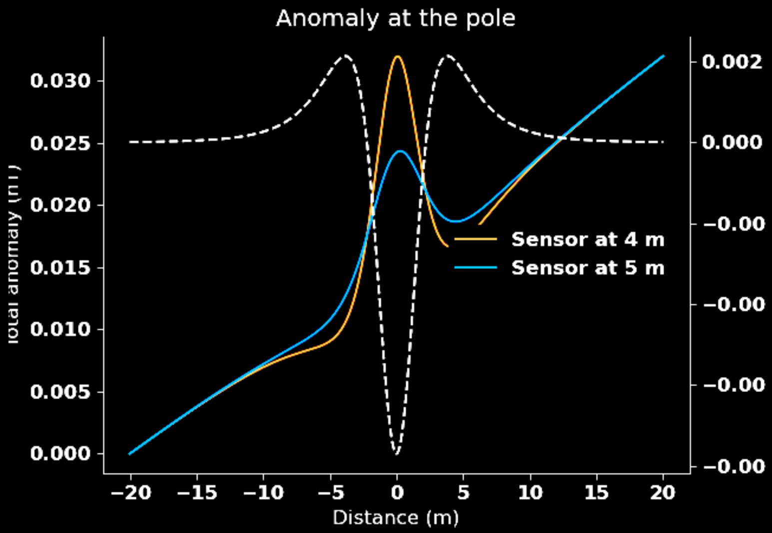
Gradiometry: Measure the vertical gradient

- Install two sensors at different heights (e.g., vertical fluxgates 0.5 m apart).
- Consider only the vertical gradient.
- Insensitive to the total field and temporal variability thereof.
- Sensitive to near-surface structures (why?).
- Sensitive to horizontal boundaries.
- Relevant for applied exercises.

Demonstration of forward model (Exercises)



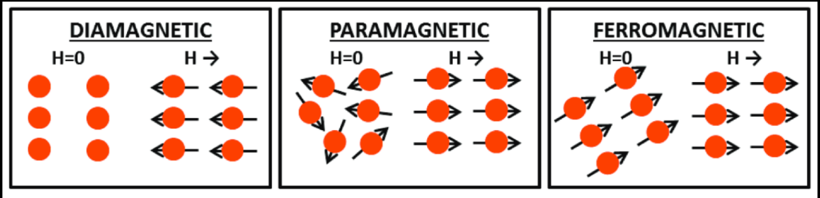
Demonstration of forward model (Exercises)



Gradiometry: Measure the vertical gradient

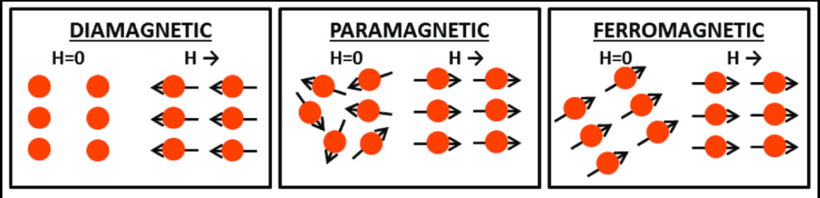
- Advantages: No basestation required
- Advantages: More sensitive to horizontal boundaries
- Advantages: Less sensitive to large-scale patterns
- Advantage/Disadvantage: Sensitive to near-surface structures (=.
- Relevant for applied exercises.

Types of Magnetism



[Lacovacci2016]

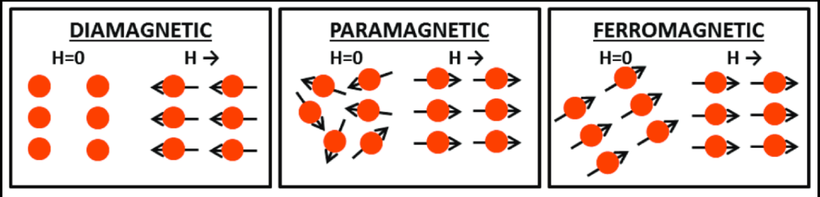
How do materials react to application of an external magnetic field?



[Lacovacci2016]

Diamagnetism

- Weak mini-dipoles induced, opposing external field ($k < 0$).
- Diappears if external field is removed.
- Occurs essentially in all materials, but is not noted everywhere, due to other effects.
- Examples: Quarzite, Calcite, ...



[Lacovacci2016]

Paramagnetism

- Material exhibits pre-existing dipoles which orient themselves in external field (unpaired electrons).
- Reinforce external field ($k > 0$).
- Effect dissipates due to thermal fluctuations once external field is removed.
- Examples: gold, copper,...

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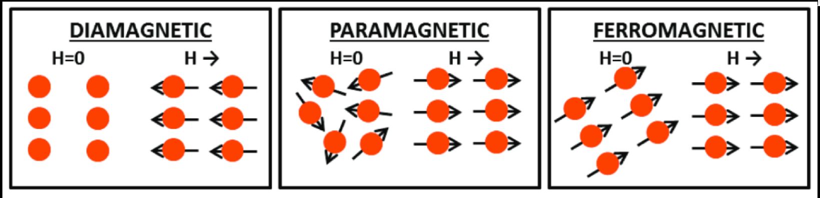

[Lacovacci2016]

Ferromagnetism

- Material exhibits pre-existing dipoles which are oriented in domains with strong coupling.
- Domains align and strongly reinforce external field ($\chi >> 0$).
- Effect can be maintained if external field is removed.
- Examples: iron, nickel, ...

Applications: Long-wavelength mapping

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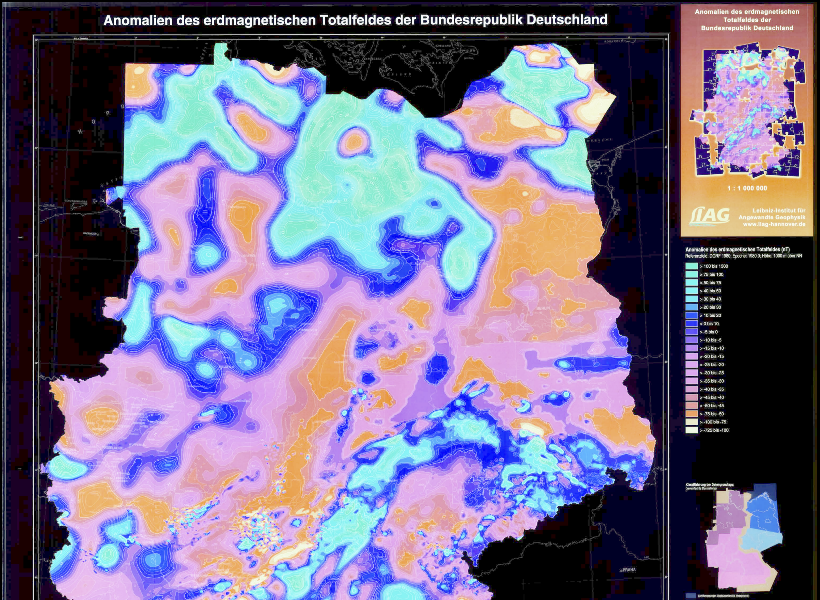
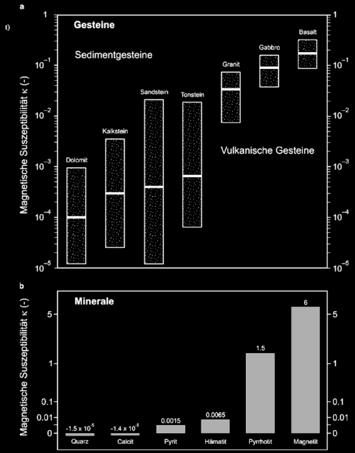


Understanding **induced** magnetic anomalies, halfwidth criteria, proton precission (Overhauser), fluxgate (Förster), gradiometer (often Förster), induction, para-, dia-, and ferromagnetism and its relation to magnetic susceptibility.

Learning goals:

- Magnetism in rocks
- Remanence
- Applications in Paleomagnetics

Magnetism in rocks



Units nT; Reference:

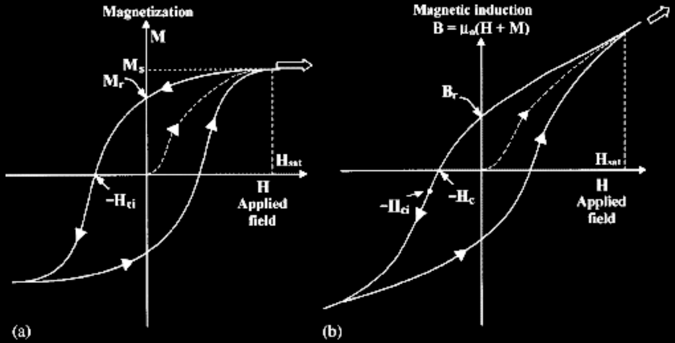
2007, Lowrie

Induced vs. Remanent Magnetization

Induced: Usually parallel to external field; it disappears if external field disappears; quantified by susceptibility (para-, dia- magnetisme)

Remanent: Orientation dependent on history; temporally persistent; occurs in ferromagnetic materials with large magnetic domains.

Definition of remanent magnetization / hysteresis

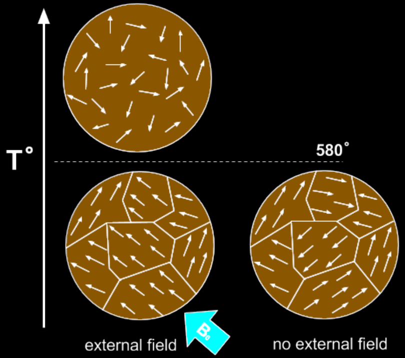


Reference:

Sung & Rudowicz, J. Magnetism and Magnetic Mat., 2003

- Thermoremanent magnetization: cooling below Curie Temperature in igneous Rocks
- Detrital magnetization: slow setteling of fine-grained materials (clay)
- Lightning Strike induced: (should occur within lighting belt)
- ....

Curie Temperature 1



Units nT; Reference:

2017, GeoSci Developers: CCA4.0 Int. Lic.

Curie Temperature 2

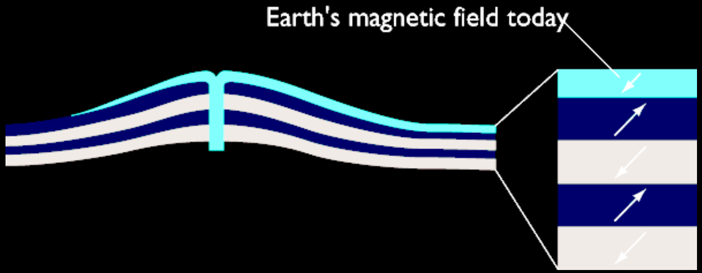
https://www.youtube.com/watch?v=haVX24hOwQI

Koenigsberger ratio

$$Q = \frac{M_r}{M_i}$$

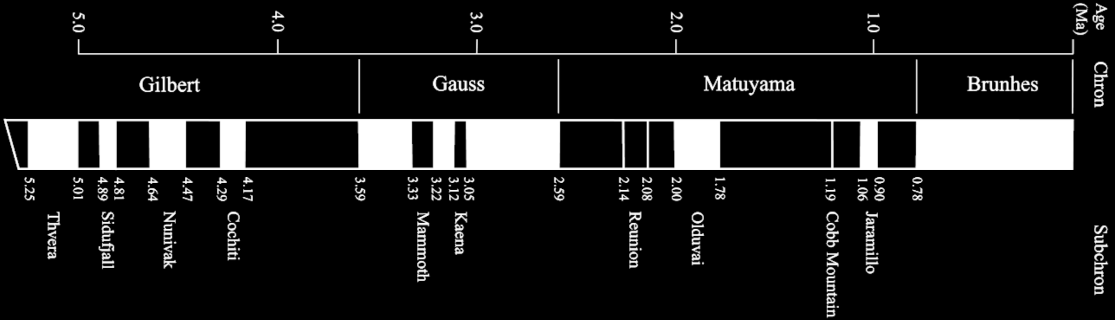
For some materials this ratio can be very large, i.e., virtually independent of the current Earth magnetic field. This is foundational for the field of paleomagnetics.

Paleomagnetic dating



- Magnetization parallel or anti-parallel to the poles
- Either rocks or field has reversed
- Use as timescale once externally dated

Paleomagnetic dating



- In some vertically stacked sections samples the remanent magnetization is not (anti-)parallel to the poles.
- Polar wandering?
- Continental wandering?
- How can we differntiate between both hypothesis?