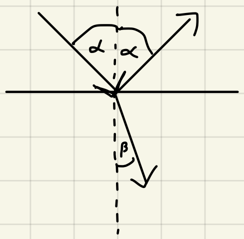
VL 3, 23.4.24

Lernziele

1. Akkustische Wellen an (horizontalen) Grenzflächen [konzeptuell]
2. Charakteristische seismische Messung [konzeptuell]
3. $\boxed{\text{Refraktionsseismic: horizontale Zweischichtfall [quantitativ]}}$ Klausur

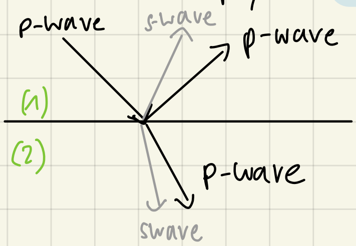
1) Waves at interfaces



$$\boxed{\frac{\sin(\alpha)}{\sin(\beta)} = \frac{V_1}{V_2}}$$

"oblique" incedence

additional "physics": mode conversion



snells law only applies to the same modes (p,s)

Reflection Transmission coefficients

$$\boxed{R = \frac{\rho_2 V_2 - \rho_1 V_1}{\rho_2 V_2 + \rho_1 V_1} = \frac{I_2 - I_1}{I_2 + I_1} \qquad R \in [-1;1]}$$

← "normal" incedence

ρ: density
v: akk velocity

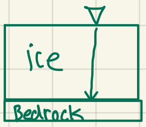
I: seismic impedence

if $R = 1$: no transmission!
$R = -1$: no transmission!

The sign of R indicates existence or absence of polarity/phase reversal

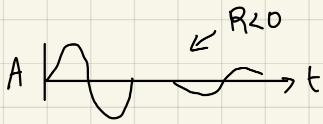
conceptual question exam

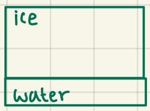
Conceptually



no reversal

impedence increases with depth (normally)



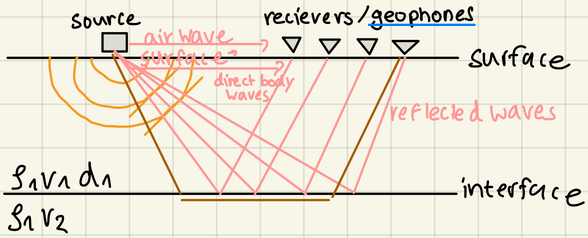


reversal

Energy requires a relationship btw reflection and transmission:

$$\boxed{|R| + |T| = 1}$$

2) Characteristic seismic shot record:



1. $V_{air} = 330\ ms^{-1}$ p-wave
2. surface waves (Reyl./Love)
3. reflected waves (p/s)
4. direct body waves (p/s)
5. Headwave/Mintrop wave ("Kopfwelle")

Headwave

$$\frac{\sin(i)}{\sin(o)} = \frac{V_1}{V_2}$$

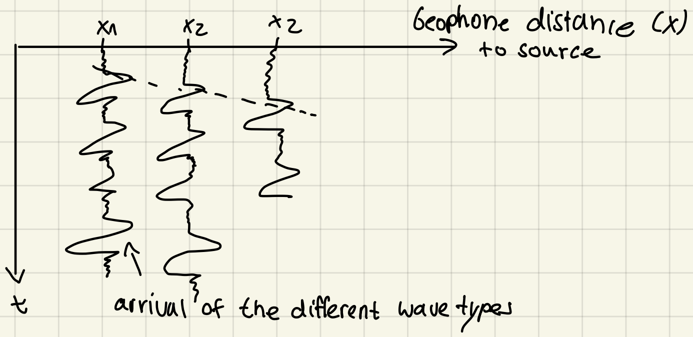
$$\boxed{\sin(i_c) = \frac{V_1}{V_2}}$$

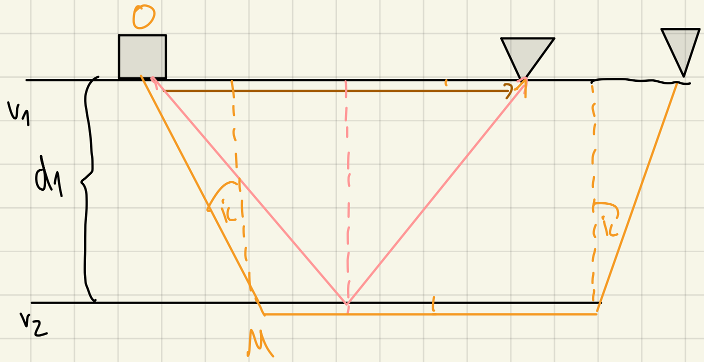
What if $o = 90°$? → critical refraction

The headwave travels below the boundary (as a body wave) with velocity V_2 and it radiates waves back to surface at the critical angle i_c (comparatively weak amplitudes)

only happens if seismic velocity increases with depth
$V(z)$ increases with depth

Shot record





1. direct body wave $\quad t(x) = x/V_1$

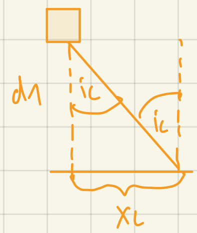
2. reflected wave $\quad t(x) = 2 \cdot \sqrt{\frac{x^2}{4} + d_1^2} \cdot \frac{1}{V_1}$

$$t(x=0) = \frac{2d_1}{V_1}$$

3. Headwave

$$t(x) = 2\,\frac{\overline{OM}}{V_1} + \frac{\overline{MP}}{V_2}$$

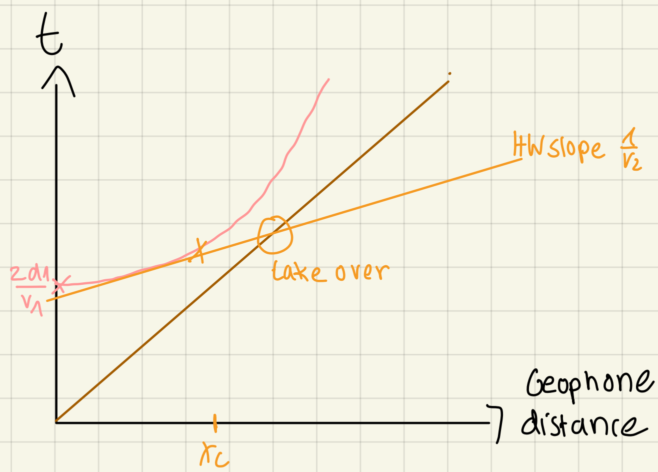
What is $\overline{MP}$?



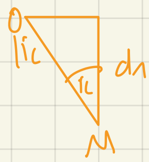
$$\tan(i_c) = \frac{x_c}{d_1}$$

$$x_c = \tan(i_c) \cdot d_1$$

$$\overrightarrow{MP} = x - 2\tan(i_c)\,d_1$$



What is $\overline{OM}$?



$$\cos(i_c) = \frac{d_1}{\overline{OM}}$$

$$\overline{OM} = \frac{d_1}{\cos(i_c)}$$

Which way is the fastest?
→ depends on distance of source
Headwave = Highway

$$t(x) = \frac{2}{V_1}\,\frac{d_1}{\cos(i_c)} + \frac{1}{V_2}\left(x - 2d_1\tan(i_c)\right)$$

$$= \frac{x}{V_2} + \frac{2d_1}{V_1\cos i_c} - \frac{2d_1\tan(i_c)}{V_2}$$

$$= \frac{x}{V_2} \quad \frac{2d_1V_2}{V_1\cos(i_c)V_2} - \frac{2d_1\sin(i_c)V_1}{V_2\cos(i_c)V_1} \qquad \sin(i_c) = \frac{V_1}{V_2}$$

$$= \frac{x}{V_2} \quad \frac{2d_1\left(V_2 - \frac{V_1}{V_2}\cdot V_1\right)}{V_1V_2\,\cos(i_c)}$$

$$= \frac{x}{v_2} \frac{2d_1\left(v_2 - \frac{v_1^2}{v_2}\right)}{v_1 v_2 \sqrt{1-\sin^2(i_c)}}$$

$$= \frac{x}{v_2} + \frac{2d_1}{v_1 v_2}\left(\frac{v_2 - \frac{v_1^2}{v_2^2}}{\sqrt{1-\frac{v_1^2}{v_2^2}}}\right)$$

$$= \frac{x}{v_2} + \frac{2d_1}{v_1 v_2}\left(\frac{v_2\left(1-\frac{v_1^2}{v_2^2}\right)}{\sqrt{1-\frac{v_1^2}{v_2^2}}}\right)$$

$$= \frac{x}{v_2} + \frac{2d_1}{v_1 v_2} v_2 \sqrt{1-\frac{v_1^2}{v_2^2}}$$

$$\boxed{t(x) = \frac{x}{v_2} + \frac{2d_1}{v_1} \cos(i_c)}$$

$$= \frac{x}{v_2} + \frac{2d_1}{v_1} \frac{\sqrt{v_1^2 - v_2^2}}{v_1 v_2}$$

VL4, 25.4.24

Goals

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(1) Exercises ✓ [correct velocity]

(2) Finish theory of refractin seismics
→ N-layer case
→ dipped layers

Textbook: Clauset – Geophysics / Seismics p[276]

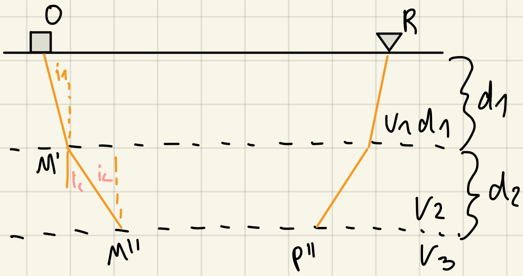
key equations so far

1. $t_{d, p_1} = \frac{1}{v_{p1}} x$

2. $t_{r, p_1} = \frac{2}{v_{p1}} \sqrt{\frac{x^2}{4} + d_1^2}$

3. $t_{HW, p} = \frac{1}{v_{p_2}} x + \underbrace{\frac{2 d_1}{v_1} \cos(i_c)}_{2 d_1 \frac{\sqrt{v_2^2 - v_1^2}}{v_1 v_2}}$

Horizontal two layer case



$\cos(i_c) = \frac{d_2}{\overline{M'M''}} \Leftrightarrow \overline{M'M''} = \frac{d_2}{\cos(i_c)}$

i_1: normal refraction $\frac{\sin(i_1)}{\sin(i_c)} = \frac{v_1}{v_2}$ (snells law)

i_c: critical refraction $\sin(i_c) = \frac{v_2}{v_3}$

$$t = \frac{2\overline{OM'}}{v_1} + \frac{2\overline{M'M''}}{v_2} + \frac{\overline{M''P''}}{v_3}$$

$$= \frac{2d_1}{v_1 \cos(i_1)} + \frac{2d_2}{v_2 \cos(i_c)} + \frac{x - 2d_1 \tan(i_1) - 2d_2 \tan(i_c)}{v_3}$$

$\tan = \frac{\sin}{\cos}$

$$= \frac{x}{v_3} + \frac{2d_1}{v_1 \cos(i_1)} + \frac{2d_2}{v_2 \cos(i_{2,c})} - \frac{2d_1 \sin(i_1)}{v_3 \cos(i_1)} - \frac{2d_2 \sin(i_{2,c})}{v_3 \cos(i_{2,c})}$$

$$t = \frac{x}{v_3} + \frac{2d_2}{v_2 \cos(i_{2,c})} \underbrace{\left(1 - \underbrace{\frac{v_2}{v_3}}_{\sin(i_{2,c})} \sin(i_{2,c})\right)}_{= 1 - \sin^2(i_{2,c}) = \cos^2(i_{2,c})} + \frac{2d_1}{v_1 \cos(i_1)} \underbrace{\left(1 - \underbrace{\frac{v_1}{v_3}}_{\sin(i_1)} \sin(i_1)\right)}_{1 - \sin^2(i_1) = \cos^2(i_1)}$$

$\frac{\sin(i_1)}{\sin(i_c)} = \frac{v_1}{v_2} \Rightarrow$ with $\sin(i_c) = \frac{v_2}{v_3}$

$\sin(i_1) = \frac{v_1}{v_2} \frac{v_2}{v_3} = \frac{v_1}{v_3}$

$$t = \frac{x}{V_3} + \frac{2d_1}{V_1}\cos(i_1) + \frac{2d_2}{V_2}\cos(i_{2,c})$$

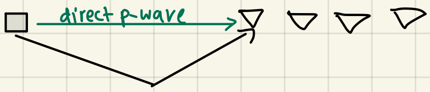
for comparison 1 layer case: $t_{HW} = \frac{1}{V_2}x + \frac{2d_1}{V_1}(\cos i_c)$

$\Rightarrow$ Generalization to N- (horizontal) layers

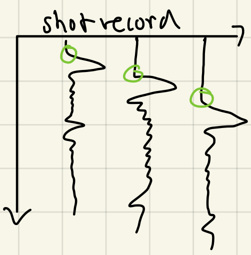
$$\boxed{t_{HW_N} = \frac{x}{v} + \sum_{k=1}^{N-1} \frac{2d_k \cos(i_k)}{v_k}}$$

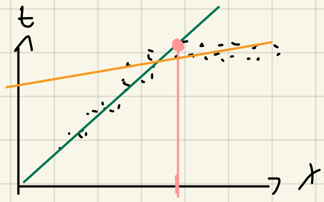
Template for horizontal refraction seismics

1. collect data



2. pick the first arrival !





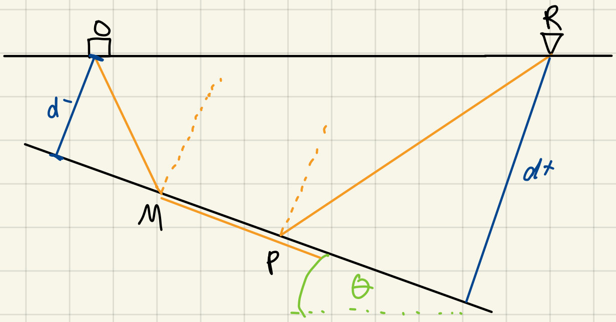
3) identify knick points

4) linear regression for each section (first one through origin)

5) Get the velocities from $(v_1, v_2, \dots)$ from slope $m = \frac{1}{v}$

6) Get thickness from y-intercept (HW)

Dipped layers



$$t^- = \frac{\overline{OM}}{v_1} + \frac{\overline{MP}}{v_2} + \frac{\overline{PR}}{v_1}$$

$$= \frac{d^-}{v_1 \cos(i_c)} + \frac{d^+}{v_1 \cos(i_c)} + \frac{x \cdot \cos(\theta) - d^- \tan(i_c) - d^+ \tan(i_c)}{v_2}$$