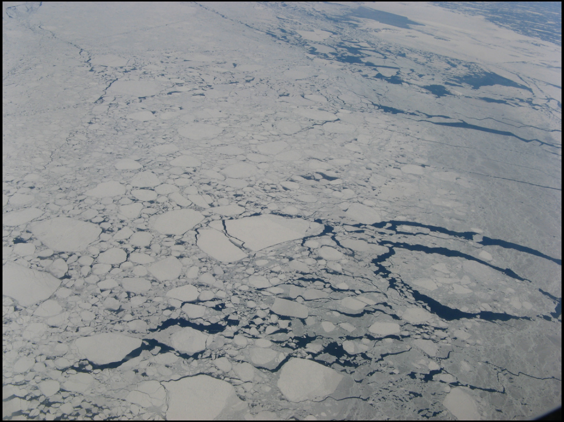
Introduction to applied Geophysics

R. Drews

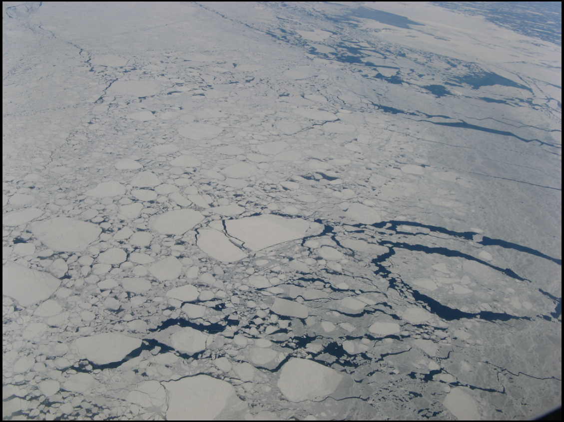
Induction Method

Problem: Sea-ice thickness



Reference:

Problem: Sea-ice thickness



Reference:

Nipponianinippon (Wikipedia) CC-BY SA 2.0.

Problem: Sea-ice thickness

- Sea-ice provides a (strong) internal Earth System feedback via albedo. It is a major player for formation and decay of Pleistocene ice sheets.
- Decreasing sea-ice thickness and extent has commercial applications in, e.g., ship navigation (cf. drift & noise)
- [Video Dr. Steffi Arndt]
- Problem: How can we determine sea ice thickness in space (and time)?

Problem: Sea-ice thickness

- Electrical conductivity of ice and ocean water are hugely different.
- Resistivity mapping will not give us spatial coverage (ground-coupled)
- A solution without cables and ground-coupling is required!
- Electromagnetic induction can do this.

Learning Goals

- Understand the principle of EMI (qualitatively)
- Understand basics of R-L circuits with low-frequency AC (quantiatively)
- Tools: Oscillations, complex numbers,...

Online Ressources

- em.geosci.xyz
- lecture textbooks (different approaches exist.)

Physics of electromagnetic induction (EMI)

- Electrical conductivity of ice and ocean water are hugely different.
- Resistivity mapping will not give us spatial coverage (ground-coupled)
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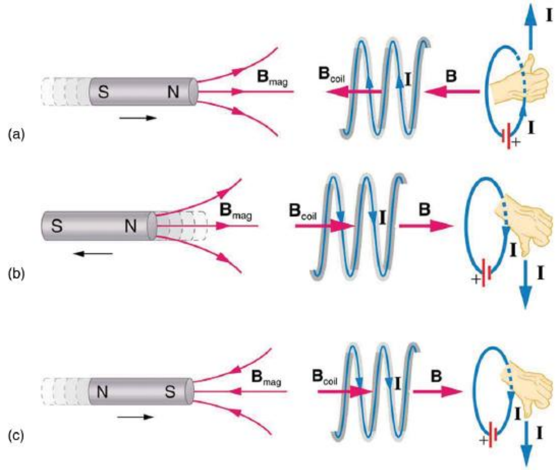
- Electromagnetic induction can do this.

Physics of electromagnetic induction (EMI): Faraday's Law



Reference:

Physics of electromagnetic induction (EMI): Faraday's Law

$$\nabla \times \vec{E}(\vec{x}, t) = -\frac{\partial}{\partial t}\vec{B}(\vec{x}, t) \quad (1)$$

$$\oint_{\partial\Sigma} \vec{E}(\vec{x}, t) \cdot d\vec{l} = -\int_{\Sigma} \frac{\partial}{\partial t}\vec{B}(\vec{x}, t) \cdot d\vec{A} \quad (2)$$

Both formulations are equivalent. Both look scary (and they are!), but they become intuitive for *simple* geometries.

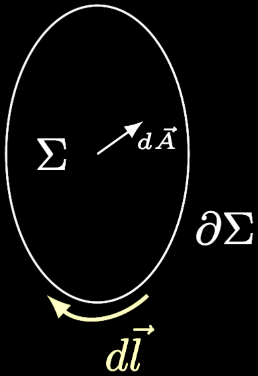
Physics of electromagnetic induction (EMI): Faraday's Law

$$\nabla \times \vec{E}(\vec{x}, t) = -\frac{\partial}{\partial t}\vec{B}(\vec{x}, t) \tag{3}$$

$$\oint_{\partial\Sigma} \vec{E}(\vec{x}, t) \cdot d\vec{l} = -\int_{\Sigma} \frac{\partial}{\partial t}\vec{B}(\vec{x}, t) \cdot d\vec{A} \tag{4}$$

In the following we focus exclusively on the principle and describe the subsurface with standard parts of electric circuits. This is enough for our introduction.

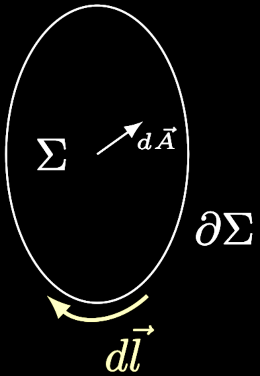
$$\underbrace{\oint_{\partial\Sigma} \vec{E}(\vec{x},t)\cdot d\vec{l}}_{\text{Line integral over rim.}} = -\int_{\Sigma} \frac{\partial}{\partial t}\vec{B}(\vec{x},t)\cdot d\vec{A}$$



Physics of electromagnetic induction (EMI): Faraday's Law

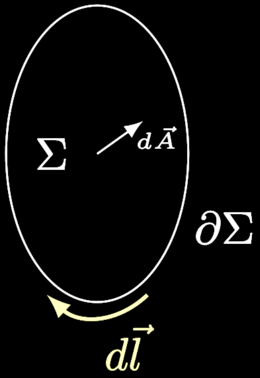
$$\underbrace{\oint_{\partial\Sigma} \vec{E}(\vec{x},t)\cdot d\vec{l}}_{} \qquad = -\int_{\Sigma} \frac{\partial}{\partial t}\vec{B}(\vec{x},t)\cdot d\vec{A}$$

EMF (V): measure for energy transfer (no cables!) into the circuit.



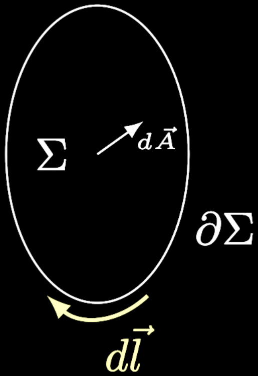
Physics of electromagnetic induction (EMI): Faraday's Law

$$\oint_{\partial\Sigma} \vec{E}(\vec{x}, t) \cdot d\vec{l} = \underbrace{-\int_{\Sigma} \frac{\partial}{\partial t} \vec{B}(\vec{x}, t) \cdot d\vec{A}}_{\text{Area integral}}$$



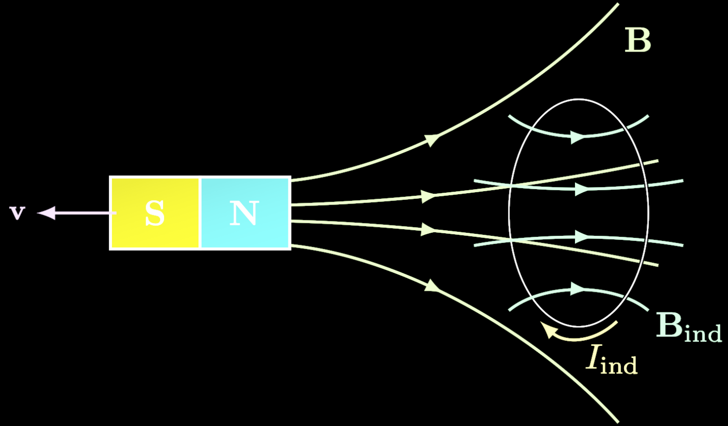
Physics of electromagnetic induction (EMI): Faraday's Law

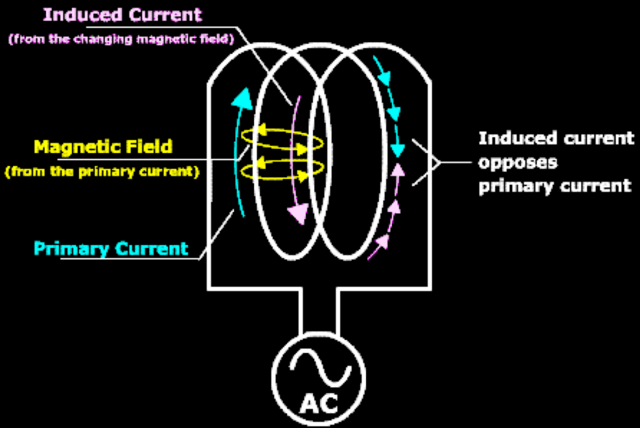
$$\oint_{\partial\Sigma} \vec{E}(\vec{x}, t) \cdot d\vec{l} = \underbrace{-\int_{\Sigma} \frac{\partial}{\partial t}\vec{B}(\vec{x}, t) \cdot d\vec{A}}_{\text{Magnitude of intercepted magnetic flux (direction dependent!)}}$$



Physics of electromagnetic induction (EMI): Faraday's Law

$$\oint_{\partial\Sigma} \vec{E}(\vec{x}, t) \cdot d\vec{l} = -\int_{\Sigma} \frac{\partial}{\partial t} \vec{B}(\vec{x}, t) \cdot d\vec{A}$$





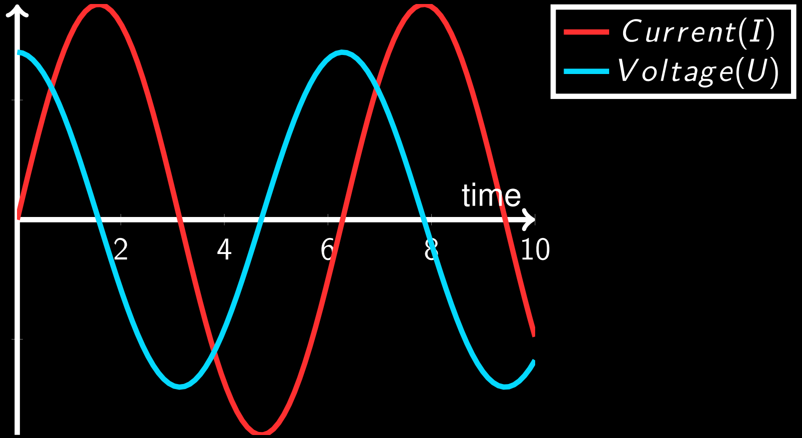
If magnetic field is caused by own current in circle: $U(t) = -L\frac{dI}{dt}$

L is the inductances describing material coil characteristics (i.e. number of loops).

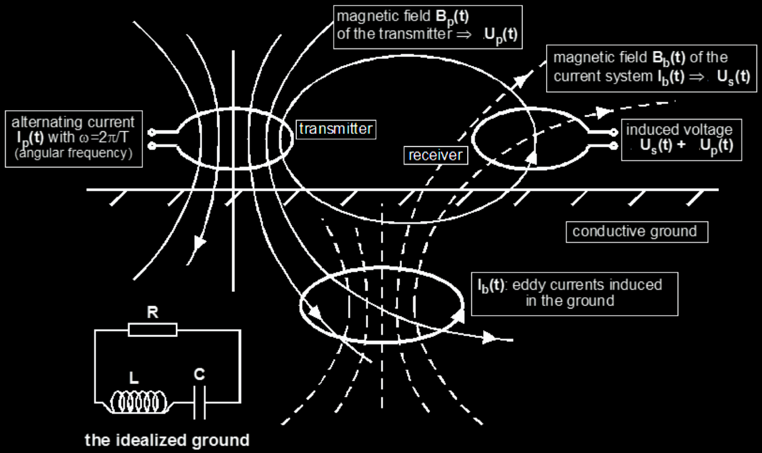
Assume that AC current is: $I(t) = I_p \sin(\omega t)$
Show that U is phase shifted by 90 degrees and that the involved amplitudes are frequency dependent. How is that differences from a 'normal' R-circuit?

Self-Inductance

$$U(t) = -L\frac{d}{dt}I = -L\frac{d}{dt}(I_p \sin(\omega t)) = -LI_p\omega\cos(\omega t) = -LI_p\omega\sin\left(\omega t + \frac{\pi}{2}\right))$$



EMI in geophysics



Note: We will consider the sub-surface as an R-L circuit.

EMI analysis the ratio of secondary and primary voltage in receiver coil.

EMI analysis the signal also in terms of phase shifts (in-phase, quadrature)

$$\frac{U_s}{U_p} = -\frac{L_{23}L_{12}}{L_{l2}L_{13}}\left(\frac{1}{1+\alpha^2}(\alpha^2 + i\alpha)\right)$$

$$\alpha = \omega\frac{L_{l2}}{R_{l2}}$$

Where does this come from? Derive on Blackboard.

We control the current in the primary circuit (indexed with loop 1). We choose the notation of complex exponentials to describe the *oscillating* AC input current in loop 1 I_{l1}:

$$I_{l1}(t) = I_1 e^{i\omega t} = I_1(\cos(\omega t) + i \sin(\omega t))$$

$\rightarrow$ This will initiate a time variable B field (Ampère-Maxwell law $\nabla \times H = j + \frac{\partial D}{\partial t}$)

$\rightarrow$ This B field intersects loop 2 (sub-surface) and lope 3 (receiver).

$\rightarrow$ Which voltage is induced at in loops 2 and 3 because of loop 1?

Using Faraday's law:

$$U_{l2} = -L_{12}\frac{dI_{l1}}{dt} = i\omega L_{12} I_1 e^{i\omega t}$$

L_{13} is hereby a coupling factor that accounts for the relative orientation between the loops (e.g., incl. loop orientation.)
The same holds for the induced potential at loop 3 which we call U_p for *primary* potential:

$$U_p = -L_{13}\frac{dI_{l1}}{dt} = -i\omega L_{13} I_1 e^{i\omega t}$$

Which current's do the induced potentials in loops 2 and 3 drive? Ohm's law will tell us, but we need to extend it to include self-inductance for the AC case. For loop 2 we therefore define the impedance:

$$Z_{l2} = R_{l2} + i\omega L_{l2}$$

here L_{l2} is called the inductance of the sub-surface. It is a material property. Do not confuse it with the coupling factors. Now extend Ohm's law to:

$$U_{l2} = Z_{l2} \cdot \underbrace{I_{l2}}_{\text{yet unknown.}} = -i\omega L_{l2} I_1 e^{i\omega t}$$

Therefore the (eddy) current in the subsurface is:

$$\rightarrow I_{l2} = -\frac{i\omega L_{12} I_1}{R_{l2} + i\omega L_{l2}} e^{i\omega t}$$

Let us collect the sub-surface material properties in an *induction parameter* α:

$$\alpha = \omega \frac{L_{l2}}{R_{l2}}$$

so that:

$$I_{l2} = -\frac{L_{12}}{L_{l2}} \frac{i\alpha I_1}{1 + i\alpha} e^{i\omega t}$$

The two voltages induced in our receiver loop 3 are:

$$U_p = -L_{13}\frac{dI_{l1}}{dt} = -i\omega L_{13} I_1 e^{i\omega t}$$
$$U_s = -L_{23}\frac{dI_{l2}}{dt} = i\omega \frac{L_{23}L_{12}}{L_{l2}}\frac{i\alpha}{1+i\alpha} e^{i\omega t}$$

The indices s and p refer to *secondary* and *primary*, respectively.

The receiver evaluates the ratio of secondary and primary induced voltage:

$$\frac{U_s}{U_p} = -\frac{L_{23}L_{12}}{L_{l2}L_{13}}\frac{i\alpha}{1+i\alpha}$$

We can rewrite this, so that imaginary and real part become more clear (shown in exercises):

$$\frac{U_s}{U_p} = -\frac{L_{23}L_{12}}{L_{l2}L_{13}}\left(\frac{1}{1+\alpha^2}(\alpha^2+i\alpha)\right)$$

This is the key result.

$$\frac{U_s}{U_p} = -\frac{L_{23}L_{12}}{L_{l2}L_{13}}\left(\frac{1}{1+\alpha^2}(\alpha^2 + i\alpha)\right)$$

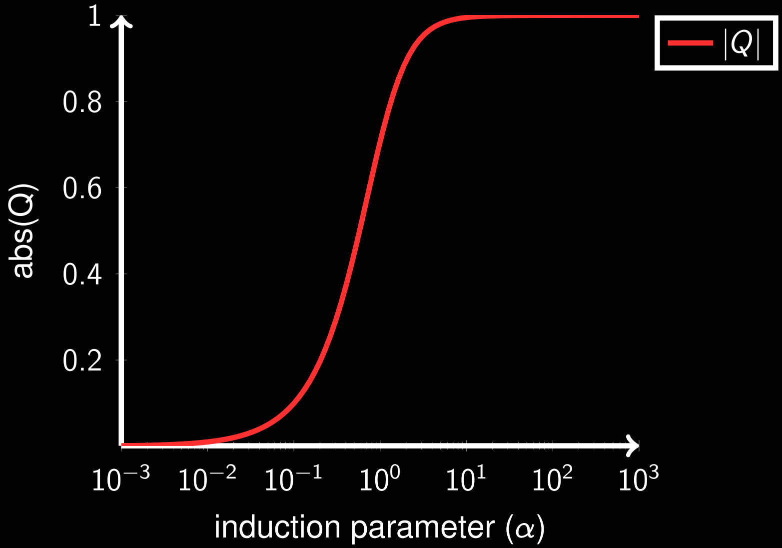
$$\alpha = \omega\frac{L_{l2}}{R_{l2}}$$

Because the transmitter sends a harmonic, continuous signal there is not much information in time. Instead, the response in the receiver is analysed in terms of frequencies and phase offset (a. k. a. FDEM - frequency domain electromagnetics.)
How can we interpret this?

This is the key result.

$$\frac{U_s}{U_p} = - \underbrace{\frac{L_{23}L_{12}}{L_{l2}L_{13}}}_{\textbf{coupling factor}} \underbrace{\left(\frac{1}{1+\alpha^2}(\alpha^2 + i\alpha)\right)}_{\text{response function } Q}$$

The ratio is a complex number. What does this mean? The magnitude of $|Q|$ tells us about the magnitude of U_s relativ to U_p. The phase angle ($\angle Q$) informs us about phase shifts between U_s and U_p. It is governed by sub-surface properties, depending on whether the resistive o the inductive part are more relevant (α).
$\rightarrow$ Exercises!


1
0.8
0.6
0.4
0.2
abs(Q)
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
induction parameter (α)
$|Q|$

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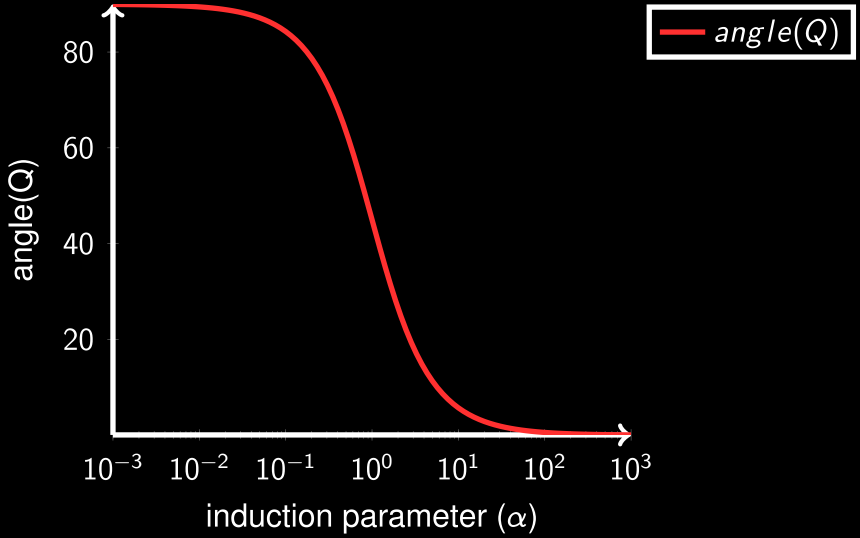
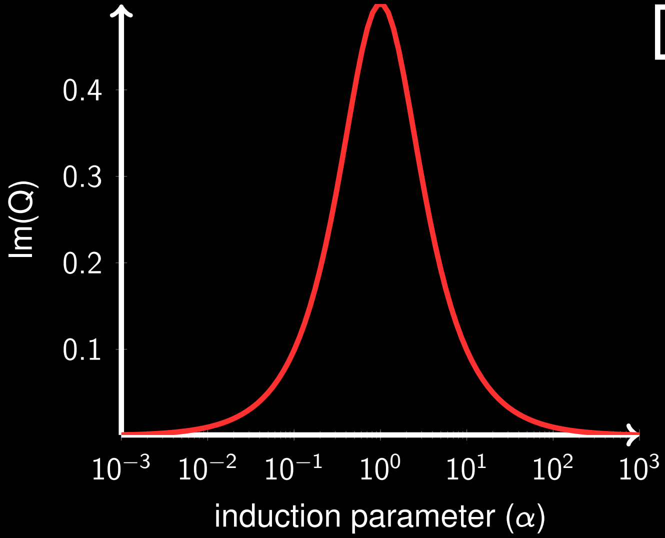
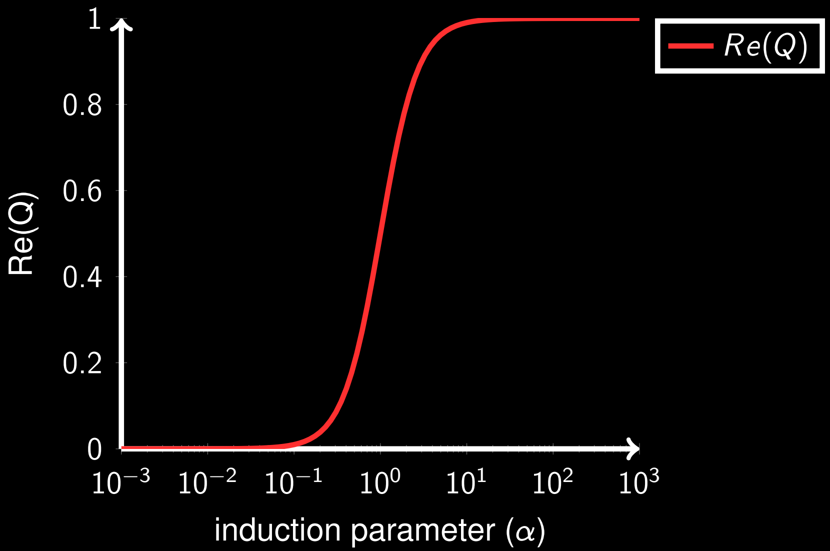




Im(Q)
0.4
0.3
0.2
0.1
10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3}
induction parameter (α)
$Im(Q)$



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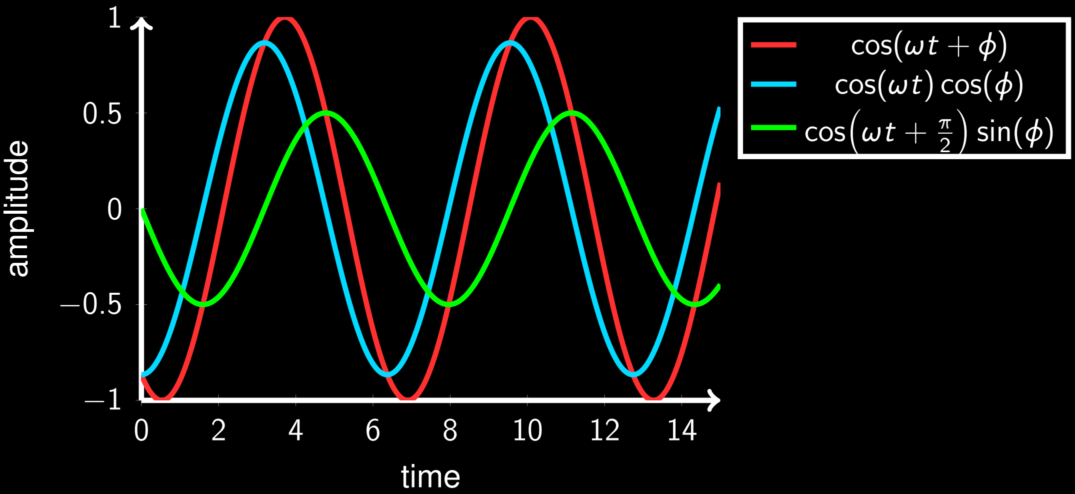
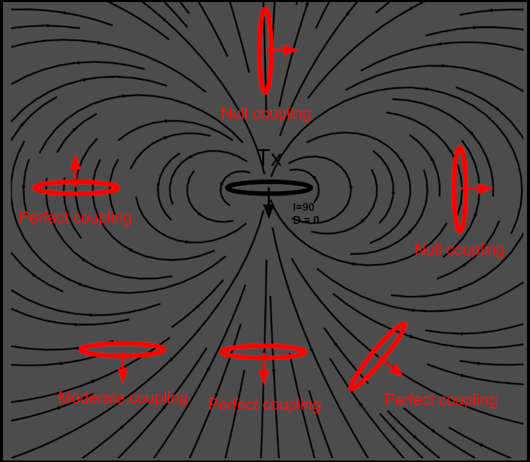
In-Phase and Quadrature Components



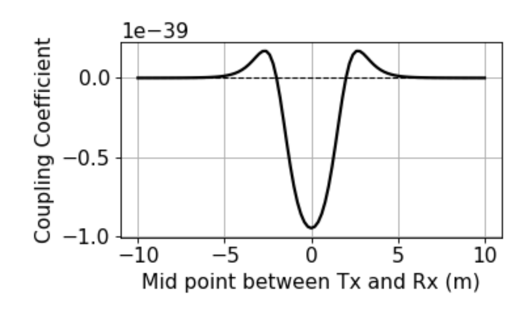
$$\cos(\omega t + \phi) = \cos(\omega t)\cos(\phi) + \cos\left(\omega t + \frac{\pi}{2}\right)\sin(\phi)$$

Application examples



Reference:

Application examples



Reference:

Resistive and conductive limits

- If $\alpha \to \infty$ we have a very good conductor and the secondary field is essentially 180° out of phase with primary field.
- If $\alpha \to 0$ very poor conductor (or a good resitor) and secondary field is 90° out-of-phase.

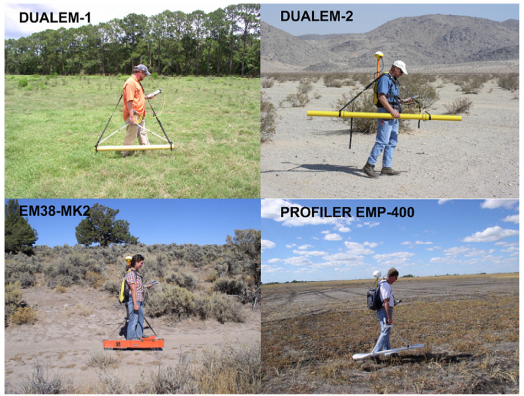
So which parameter does EMI measure?

- The three loop system is highly idealized and lends itself to ’anomaly’ hunting, but not for a rigorous derivation of the sub-surface (incl. layer thickness,...)
- The in-phase component is sensitive to the Resistivity
- The inductance L is a function of electrical (ac) conductivity.
- The inductance is also sensitive to magnetic susceptibility.

EMI there is a lot more.

- There are many ways to explore EMI data.
- The in-phase component is often sensitive to the resistivity
- The inductance L is a function of electrical (ac) conductivity which appears in the quadrature component.
- The inductance is also sensitive to magnetic susceptibility

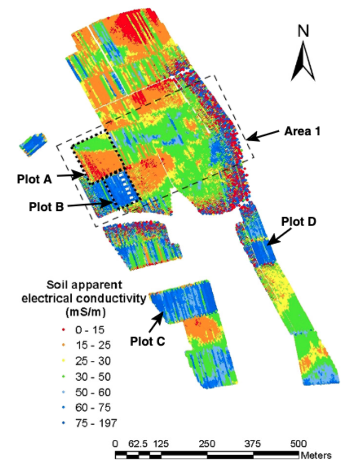
Application examples



Reference:

Doolittle & Brevik, Geoderma 223–225 (2014) 33–45

Application examples - soil moisture proxies

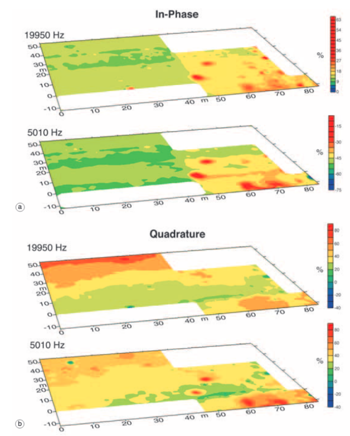


Reference:

André et al., 2012, J. Applied Geophysics

"..A particularly strong evidence of the influence of cultural operations on soil electrical conductivity is the relatively high values observed for plot B (see Fig. 1), resulting from the use of heavy machines during too wet soil conditions for the preparation of this plot to vine plantation and which induced soil compaction.."

Application examples - waste disposal sites



Reference:

Marchetti et al., Annals of Geophysic, 2002, https://doi.org/10.4401/ag-3519

Note the frequency dependent change of anomalies in quadrature map.

Application examples - sea ice

Reference:

Photo: Alfred Wegener Institute, Sea Ice Group

Application examples

Reference:

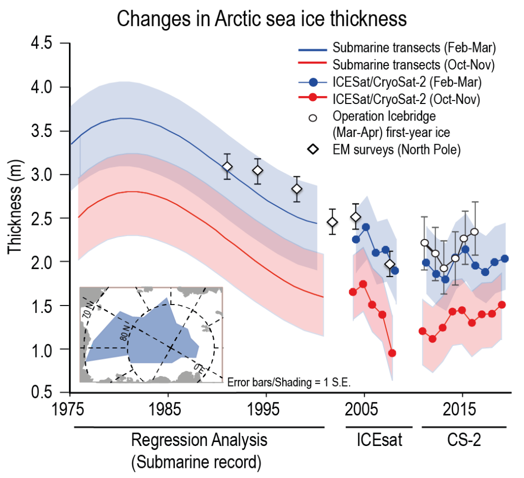
Photo: Alfred Wegener Institute, Sea Ice Group

Application examples

https://www.youtube.com/watch?v=9LYBtqo3zgQ

Reference:

Photo: Alfred Wegener Institute, Sea Ice Group



Reference:

Figure 2.21 in IPCC, 2021: Chapter 2. In: Climate Change 2021: The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change

Frequency depdendence

$$\alpha = \omega \frac{L_{l2}}{R_{l2}}$$

Why not increase the frequency by a lot to increase the signal-to-noise ratio?

$$\rightarrow$$

Explore wave propagation which will lead us into seismics.

Maxwell Equations

$$
\begin{aligned}
\nabla \cdot \vec{\mathbf{D}} &= \rho \quad \text{(Gauss)} \\
\nabla \cdot \vec{\mathbf{B}} &= 0 \quad \text{(Gauss)} \\
\nabla \times \vec{\mathbf{E}} &= -\frac{\partial \vec{\mathbf{B}}}{\partial t} \quad \text{(Faraday)} \\
\nabla \times \vec{\mathbf{H}} &= \mu_0 \vec{\mathbf{J}} + \frac{\partial \vec{\mathbf{D}}}{\partial t} \quad \text{(Ampére-Maxwell)} \\
\vec{\mathbf{D}} &= \varepsilon \varepsilon_0 \vec{\mathbf{E}} \text{(materials: electric field, dielectric field)} \\
\vec{\mathbf{H}} &= \mu \mu_0 \vec{\mathbf{B}} \quad \text{(materials: magnetizing field, magnetic induction)} \\
\vec{j} &= \sigma \vec{\mathbf{E}} \quad \text{(Ohm's law)}
\end{aligned}
$$

You don’t need to solve these equations, but remember what they mean and in which (geophysical) areas they are important.

Magnetic Method

$$
\begin{aligned}
\nabla \cdot \vec{\mathbf{D}} &= \rho \quad \text{(Gauss)} \\
\nabla \cdot \vec{\mathbf{B}} &= 0 \quad \text{(Gauss)} \\
\nabla \times \vec{\mathbf{E}} &= -\frac{\partial \vec{\mathbf{B}}}{\partial t} \quad \text{(Faraday)} \\
\nabla \times \vec{\mathbf{H}} &= \mu_0 \vec{\mathbf{J}} + \frac{\partial \vec{\mathbf{D}}}{\partial t} \quad \text{(Ampére-Maxwell)} \\
\vec{\mathbf{D}} &= \varepsilon \varepsilon_0 \vec{\mathbf{E}} \text{(materials: electric field, dielectric field)} \\
\vec{\mathbf{H}} &= \mu \mu_0 \vec{\mathbf{B}} \quad \text{(materials: magnetizing field, magnetic induction)} \\
\vec{j} &= \sigma \vec{\mathbf{E}} \quad \text{(Ohm's law)}
\end{aligned}
$$

Resistivity Method (no time dependencies)

$$
\begin{aligned}
\nabla \cdot \vec{\mathbf{D}} &= \rho \quad \text{(Gauss)} \\
\nabla \cdot \vec{\mathbf{B}} &= 0 \quad \text{(Gauss)} \\
\nabla \times \vec{\mathbf{E}} &= 0 \quad \text{(Faraday)} \\
\nabla \times \vec{\mathbf{H}} &= \mu_0 \vec{\mathbf{J}} + 0 \quad \text{(Ampére-Maxwell)} \\
\vec{\mathbf{D}} &= \varepsilon \varepsilon_0 \vec{\mathbf{E}} \text{(materials: electric field, dielectric field)} \\
\vec{\mathbf{H}} &= \mu \mu_0 \vec{\mathbf{B}} \quad \text{(materials: magnetizing field, magnetic induction)} \\
\vec{j} &= \sigma \vec{\mathbf{E}} \quad \text{(Ohm's law)}
\end{aligned}
$$

Induction Method (low-frequency time dependency)

$$
\begin{aligned}
\nabla \cdot \vec{\mathbf{D}} &= \rho \quad \text{(Gauss)} \\
\nabla \cdot \vec{\mathbf{B}} &= 0 \quad \text{(Gauss)} \\
\nabla \times \vec{\mathbf{E}} &= -\frac{\partial \vec{\mathbf{B}}}{\partial t} \quad \text{(Faraday)} \\
\nabla \times \vec{\mathbf{H}} &= \mu_0 \vec{\mathbf{J}} + 0 \quad \text{(Ampére-Maxwell)} \\
\vec{\mathbf{D}} &= \varepsilon\varepsilon_0 \vec{\mathbf{E}} \text{(materials: electric field, dielectric field)} \\
\vec{\mathbf{H}} &= \mu\mu_0 \vec{\mathbf{B}} \quad \text{(materials: magnetizing field, magnetic induction)} \\
\vec{j} &= \sigma \vec{\mathbf{E}} \quad \text{(Ohm's law)}
\end{aligned}
$$

Note: Displacement current in Ampére-Maxwell remains zero. We show later why.

Prediction of electromagntic waves

Taking the curl of Faraday and Ampere-Max results in a wave equation. This explains the existence of electromagnetic waves important, e.g., in the radar method.

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

Wave Equation

$$\underbrace{\nabla^2 \vec{E}}_{Wave} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\epsilon \underbrace{\frac{\partial^2 \vec{E}}{\partial t^2}}_{Wave}$$

Two times space, two times time gives you (undamped) waves.

Diffusion Equation

$$\underbrace{\nabla^2 \vec{E}}_{Diffusion} = \mu\sigma \underbrace{\frac{\partial \vec{E}}{\partial t}}_{Diffusion} + \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

Two times space, one time time gives you diffusion.
Compare, e.g., with Darcy's law / groundwater head.

What is the electrical permittivity?

- ✓ The magnetic permeability ($\mu = \mu_0 \mu_r$) is the response of a material to a magnetic field (dia-, para-, ferro-,...)
- – The electric permittivity ($\varepsilon = \varepsilon_0 \varepsilon_r$) is the response of a material to an electric field.

What is the electrical permittivity?

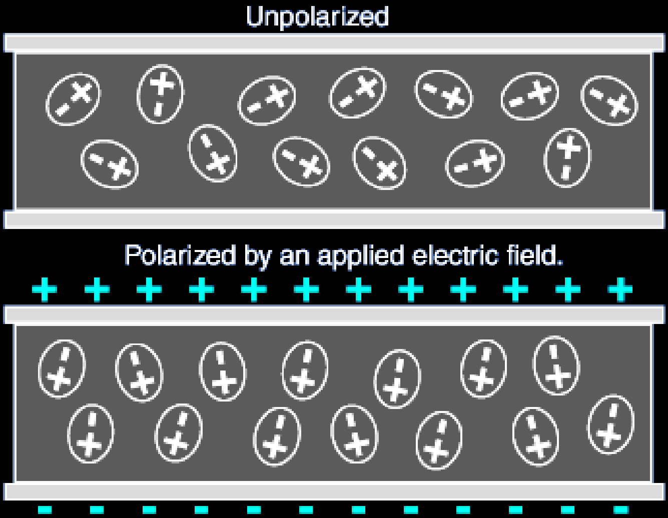


Image: Public domain.

Harmonic waves

Assume $\vec{E} = \vec{E}_0 e^{i\omega t}$

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

then straightforwardly:

$$\nabla^2 \vec{E} = i\omega\mu\sigma\vec{E} - \omega^2\mu\epsilon\vec{E}$$

Harmonic waves

Assume $\vec{E} = \vec{E}_0 e^{i\omega t}$

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

then straightforwardly:

$$\nabla^2 \vec{E} = i\omega\mu\sigma\vec{E} - \omega^2\mu\epsilon\vec{E}$$

Assume $\sigma = 10^{-4}$ S/m (resistive soil), $\varepsilon_r = 10$, $\varepsilon_0 = 8.89 \cdot 10^{-12} s^4 A^2 m^{-3} kg^{-1}$.

At which frequencies are both pre-factors of RHS approximately equal?

Assume $\vec{E} = \vec{E}_0 e^{i\omega t}$

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

then straightforwardly:

$$\nabla^2 \vec{E} = i\omega\mu\sigma\vec{E} - \omega^2\mu\epsilon\vec{E}$$

$\rightarrow f = \frac{\sigma}{2\pi\varepsilon_0\varepsilon_r} \approx =$ 180 kHz This is much higher than what is used within the induction method.
Electromagnetic waves become relevant, e.g., in the MHz range used in ground-penetrating radars.

Harmonic waves in the low-frequency approximation

$$\nabla^2 \vec{E} = \underbrace{i\omega\mu\sigma\vec{E}}_{\textbf{Damping term}}$$

or 1D:

$$\frac{\partial^2}{\partial z^2}\vec{E} = \underbrace{i\omega\mu\sigma\vec{E}}_{\textbf{Damping term}}$$

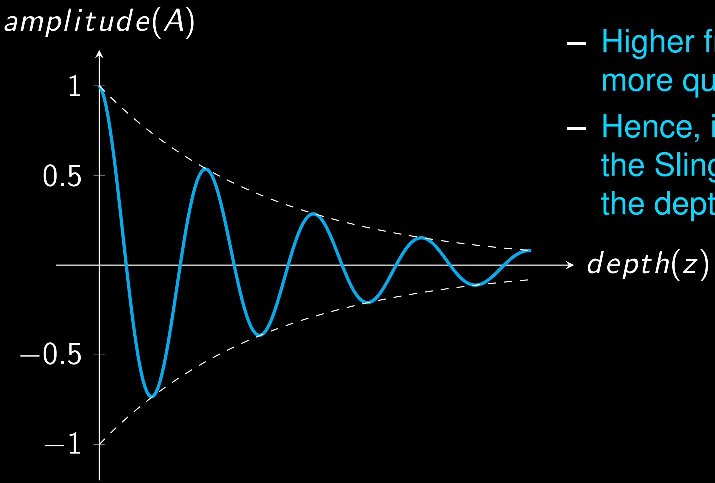
This can be solved using a damped wave:

$$E = Ae^{-kz}e^{i(\omega t - kz)}$$

with:

$$k = \sqrt{\frac{\omega\mu\sigma}{2}}$$

Skin effect



- Higher frequencies are damped more quickly.
- Hence, if we increase frequency in the Slingram method we decrease the depth penetration.

The end of induction: Word Cloud

Induction method conceptualized with three loops

Induction sensitive to (ac) electrical conductivity, resistivity, magnetic susceptibility

In-phase and out-of-phase (quadrature components)

Resistive, conductive limits.

Upper frequency barrier (tens of kHZ, both in terms of waves and in terms of attenuation)

Wave equations, damping, diffusion

Skin depth