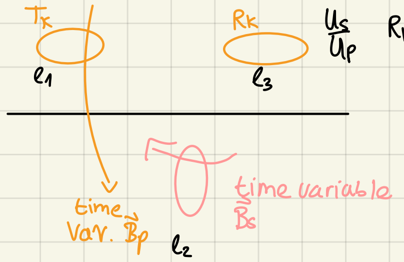
Continue derivation of response function of electromagnetic induction (EMI) using of the RL-analogy.



$\frac{U_s}{U_p}$ Rx intercepts both B_p and B_s

↑ subsurface with inductive props represended by "Eddy" currents sensitive to conductivity

$$I_{e2} = \frac{i\omega L_{12} I_1}{R_{e2} + i\omega L_{e2}} e^{i\omega t} = \frac{L_{12}}{L_{e2}} \frac{i\alpha I_1}{1+i\alpha} e^{i\omega t} = \frac{L_{12}}{L_{e2}} \frac{i\alpha I_1}{1+i\alpha} e^{i\omega t}$$

$$\alpha = \omega \frac{L_{e2}}{R_e}$$

$$U_p = -L_{13} \frac{dI_1}{dt} = -L_{13}\, i\omega I_1 e^{i\omega t}$$

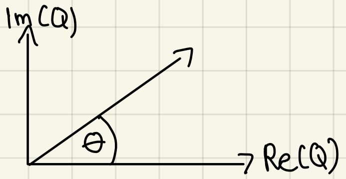
$$U_s = -L_{23} \frac{dI_{e2}}{dt} = i\omega \left[\frac{i\omega L_{12} I_1}{R_{e2} + i\omega L_{e2}} \right] e^{i\omega t}$$

$$\frac{U_s}{U_p} = -\frac{L_{23} L_{12}}{L_{e2} L_{13}} \left[\frac{1}{(1+\alpha^2)} (a^2 + i\alpha) \right]$$

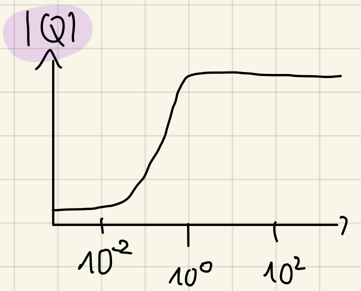
↑ real and imagenery parts are clear

- Derivation is straight forward but a bit fiddely
- $\frac{U_s}{U_p}$ at Rx depends on geometry factor $\frac{-L_{23} L_{12}}{L_{e2} L_{13}}$
- $\frac{U_s}{U_p}$ at Rx depends complex $Q = \frac{1}{(1+\alpha^2)} (a^2 + i\alpha)$

For the interpretation it is useful to vizualize $|Q|$ or $\angle Q$ (phase angle).



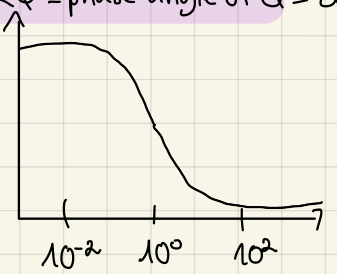
$|Q|$: Calculate or look it up $\qquad |Q| = \left| \frac{\alpha^2 + i\alpha}{\alpha^2 + 1} \right|$



$$\alpha = \omega \frac{L_{e2}}{R_e}$$

- if α is large we are in the conductive limit
  → U_s is large; much signal from subsurface
- if α is small (e.g. R_e large or $\omega = 0$) we are in the resistive limit and there is little signal from the subsurface

$\angle Q$ = phase angle of $Q = \arctan\left(\frac{Im(Q)}{Re(Q)}\right)$



The phase angle of Q provides information of the phasing between U_s and U_p.

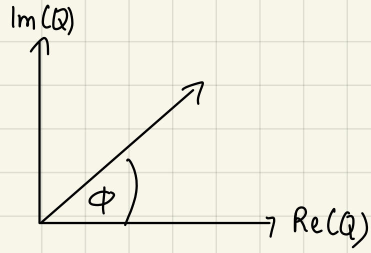
Continuation with EMI

The phase angle $\angle Q$; angle(Q) provides information about the phasing between $\frac{U_s}{U_p}$.
What is the phase angle of Q?

$$Q = \frac{\alpha^2 + i\alpha}{a^2+1}$$

$$\tan(\theta) = \frac{Im}{Re} = \frac{\alpha}{\alpha^2} = \frac{1}{\alpha}$$

$$= \arctan\left(\frac{1}{\alpha}\right)$$



More interesting is phase difference btw U_s and U_p

$$U_s = -L_{23}\, i\omega \frac{L_{12}}{L_{22}} \frac{i\alpha}{(1+i\alpha)} e^{i\omega t}$$

$$= C i \frac{\alpha^2 + i\alpha}{1+\alpha^2} = C\left(\frac{-\alpha + i\alpha^2}{1+\alpha^2}\right)$$

How turn in proper complex number:

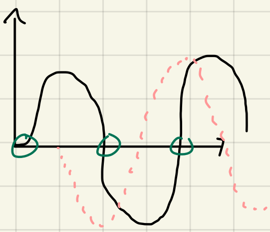
$$\frac{i\alpha}{(1+i\alpha)} \frac{(1-i\alpha)}{(1-i\alpha)} = \frac{i\alpha(1-i\alpha)}{1+\alpha^2} = \frac{\alpha^2+i\alpha}{1+\alpha^2}$$

$$\text{angle}(U_s) = \angle U_s = \tan\left(\frac{\frac{+\alpha^2}{1+\alpha^2}}{\frac{-\alpha}{1+\alpha}}\right) = \tan(-\alpha) = \tan^{-1}\left(\omega\frac{L_e}{R}\right) \Rightarrow \left[0, \frac{\pi}{2}\right]$$

↖ induction parameter

$$\text{angle}(U_p) = \angle U_p = \frac{\pi}{2}$$

Conceptual meaning: · U_p is $\frac{\pi}{2}$ offset at R_x relative to T_x (b/c induction current is largest when chang of $\vec{B}$ is largest)



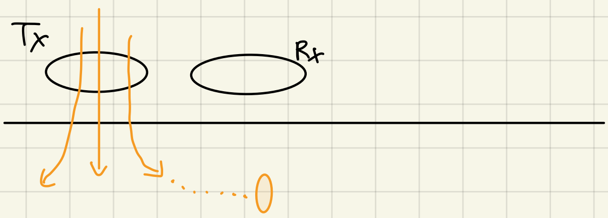
· U_s is also offset but the phase offset depends on subsurface properties
(inductive vs resistive limits)
$\frac{\pi}{2}$ $\quad$ 0

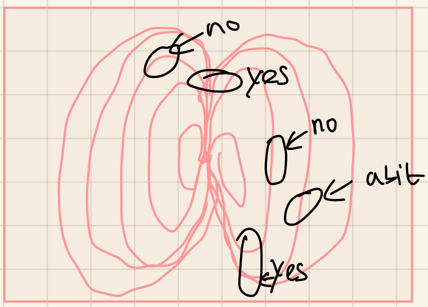
=> Phase shift can be used to characterize the subsurface and also to investigate ω dependency.

Typical / idealize EMI anomaly

In EMI the anomaly shape depends on the target and on the coupling geometry: $L_{12}; L_{23}, L_{13}$
· normal component to magnetic field? (Loop orientation + position)

Infer anomaly type for co-planar aquisition:

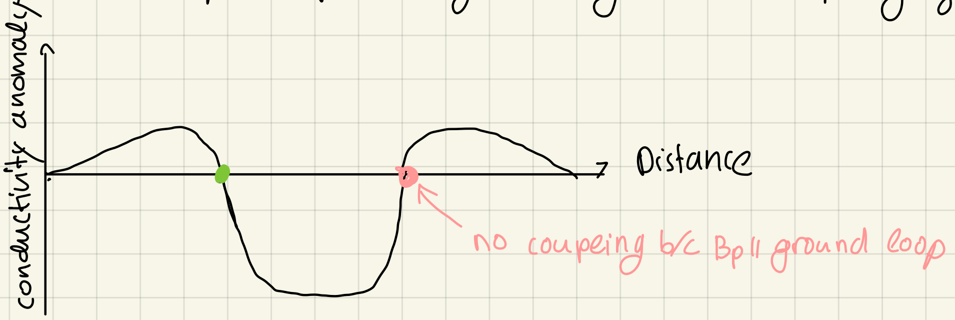




case 1: transmitter above target

Case 2: reciever above target

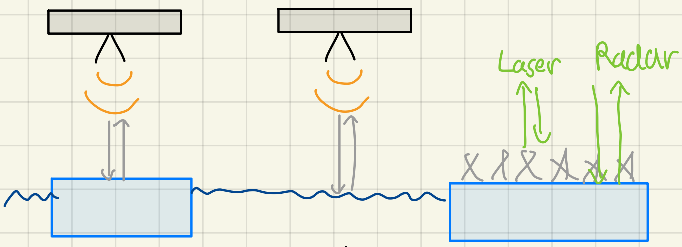
Typical shape for point target emerges from coupling geometry



Sea ice problem

- Earth System + Industry relevance is clear
- airborne EMI can measure sea ice thickness along flight tracks.
  (great! Still to little coverage to achieve objectives. However perfect for validating / calibrating global sea ice maps)

For polar coverage satellites are required, more specifically: altimeters



"Echo location principle
=> surface topography"

Ice SAT: Laser
Cryo SAT: Radar

h: Freebord
Archimedes principle will give you the thickness (of Gravimetry)